JAN 31 1927

JOINT MEETING AMERICAN PHYSICAL SOCIETY and OPTICAL SOCIETY of AMERICA, NEW YORK, FEBRUARY 25, 1927

Volume 14 Number 1

Journal of the Optical Society of America and Review of Scientific Instruments

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Published Monthly by the
OPTICAL SOCIETY OF AMERICA

OFFICE OF PUBLICATION
GEORGE BANTA PUBLISHING COMPANY
450-454 Ahnaip St.,
MENASHA, WISCONSIN

Entered as second-class matter May 31, 1921, at the postoffice at Menasha, Wis., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412, P. L. and R. authorized June 19, 1925.

Journal
of the
Optical Society of America
and
Review of Scientific Instruments

Published monthly by the Optical Society of America in cooperation with the Association of Scientific Apparatus Makers of the United States of America. The Section on Optics is devoted to original papers on experimental, theoretical, and applied optics. The Section on Scientific Instruments is devoted to original papers on instruments of all kinds (electrical, mechanical, thermal, etc., as well as optical) for research, instruction, and industrial processes. Manuscripts for publication are invited, whether authors are members of the Optical Society or not.

Annual subscription $5.00 dating from January of current year. Single copies $0.60. Back numbers: Vols. 1–5 (The Journal of the Optical Society, 1917–1921 inclusive) $16.00. Vol. 6–7, $5.00 per volume; Vol. 8—, $2.50 per volume. Address publishers, "George Banta Publishing Company, 450–454 Ahnaip Street, Menasha, Wisconsin, or F. K. Richtmyer, Manager, Cornell University, Ithaca, New York," for all correspondence regarding subscriptions, back numbers, changes of address, advertising rates and accounts, and missing numbers.

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of the
Optical Society of America
and
Review of Scientific Instruments

Vol. 14 JANUARY, 1927 Number 1

ACTION OF A LIGHT-SOURCE IN THE FIELD OF VIEW IN LOWERING VISIBILITY

By L. L. Holladay[1]

In the first part of this article, results of an experimental investigation of the action of a light-source in the field of view in changing the adaptation of the fovea of the eye, and in increasing the minimum perceptible brightness difference are presented. In the second part are presented results of a study to determine whether a given light-source in the field of view is an advantage or disadvantage; and also a method is presented for expressing visibility by an exact ratio.

Part 1

Arrangement of Apparatus. The arangement of apparatus used throughout this investigation is shown in Fig. 1. The observer was stationed at a distance of 344 cm from a screen illuminated to a uniform brightness of F millilamberts by light-source L_3. The dazzle-glare-source consisted of a bowl-frosted tungsten lamp L_1 placed with its tip toward the observer within a $7'' \times 21''$ sheet iron enclosing pipe. This lamp which was placed at an angle of D degrees above the observer's line of vision gave a unidirectional component of illumination of E meter-candles at the eyes of the observer.

The test-object was in reality an annular ring of light projected upon the screen from a diffusing surface of 1.95 cm diameter illuminated by a 100-watt tungsten lamp L_2 enclosed in a 7″ sheet iron pipe as shown. The light from this small diffusing surface traversed an annular opening of 1.9 cm inside diameter and 3.8 cm outside diameter in the end of the pipe. It is apparent that the annulus as projected upon the screen was a ring with the brightness a maximum along circle A and decreasing toward each edge, as shown. The brightness of this annular test-object

[1] L. L. Holladay, J.O.S.A. & R.S.I., *12*, p. 271; 1926.

was varied by changing the voltage of lamp L_2 or by interposing one or more wire gauze filters in the path of the beam.

RESULTS OF INVESTIGATION

Contrast Sensitivity of the Eye. This preliminary investigation was for the purpose of determining the sensitivity of the eyes of the observers to brightness differences at low adaptive brightnesses such as are encountered in street lighting.

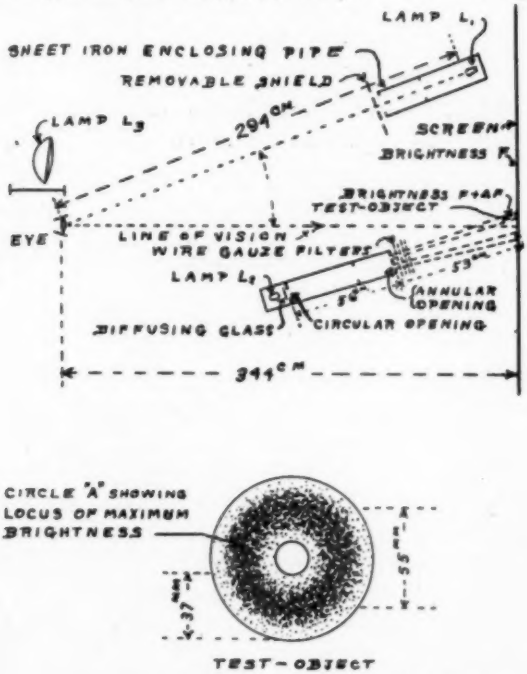

FIG. 1. *Arrangement of apparatus and test-object used.*

The method consisted in thoroughly adapting the eyes to a given brightness F of the screen and then determining the minimum brightness difference $(\Delta F)_r$, between the test-object and its background at which the annular test-object could just be seen. This minimum brightness difference, $(\Delta F)_r$, was of course the additional brightness of the annular ring projected upon the screen from lamp L_2, as described above. The average results obtained with the two observers used throughout this investigation are given in Table 1 and are shown plotted

TABLE 1. *Showing the minimum perceptible brightness difference* $(\Delta F)_T$ *in millilamberts between test-objects and background for various brightnesses F in millilamberts of the background or adaptive field. The test-object consisted of a bright annular ring with a mean diameter that subtended a visual angle of about 55 minutes and a radial width of about 37 minutes as shown in Fig. 1.*

Brightness of adaptive field in millilamberts F	Minimum perceptible brightness difference in ml $(\Delta F)_T$	Contrast sensitivity, S $F/(\Delta F)_T$
0.00002	0.0000113	1.78
0.0001	0.0000197	5.07
0.00025	0.0000444	5.63
0.0005	0.0000715	7.00
0.001	0.0000934	10.70
0.002	0.000158	12.65
0.01	0.000411	24.3
0.02	0.000775	25.8
0.1	0.00305	32.7
0.4	0.0090	44.4
0.5	0.0109	45.9
1.0	0.0192	52.2
2.0	0.0363	55.1
3.0	0.0565	53.1

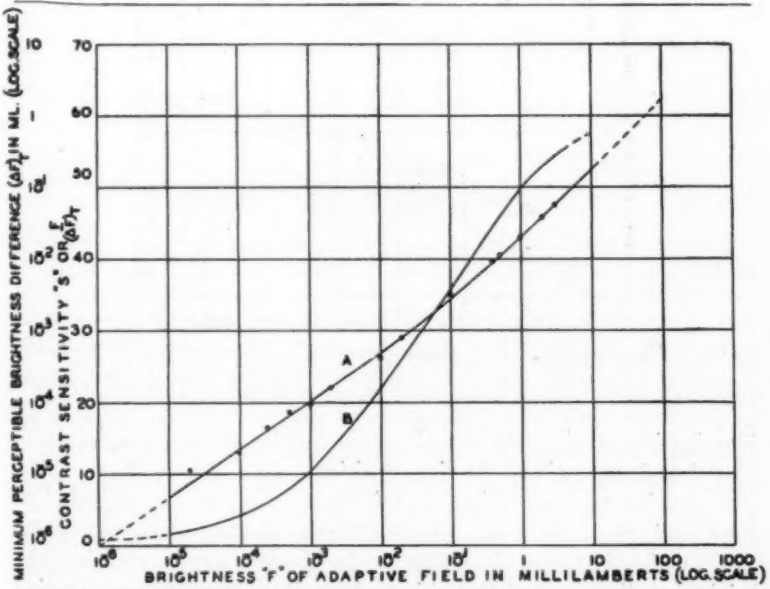

FIG. 2. *Curve A shows variation of minimum perceptible brightness difference* $(\Delta F)_T$ *with brightness F of adapting field. Curve B shows corresponding contrast sensitivity S or ratio* $F/(\Delta F)_T$

in curve *A*, Fig. 2, to logarithmic scales. In curve *B*, Fig. 2, are shown the corresponding average values of contrast sensitivity, *S*, or the ratio, $F/(\Delta F)_r$ for the eyes of the two observers.

Dazzle-Glare in Effect Increases Adaptive Brightness. The next investigation was made to determine the minimum difference in brightness $(\Delta F)_r$ between test-object and background at which test-object could just be seen when the background was of brightness *F* and the

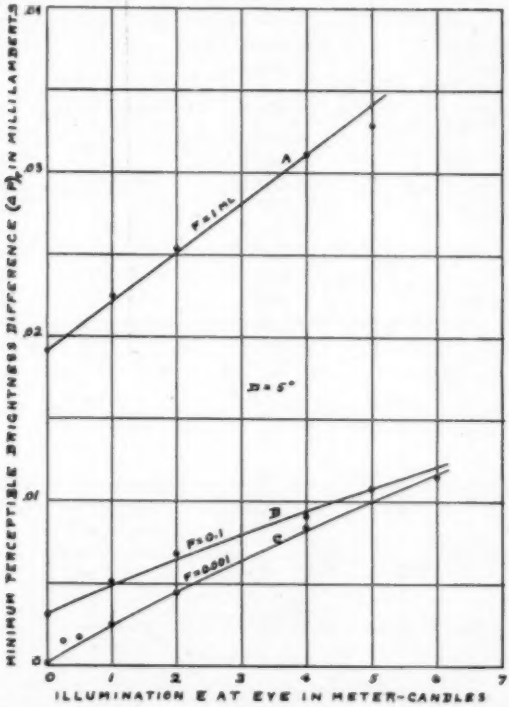

FIG. 3. *Shows variation of minimum perceptible brightness difference* $(\Delta F)_r$ *with illumination E at eye from a dazzle-source located 5 degrees above the line of vision. Curves A, B and C are for screen or background brightnesses F of 1, 0.1 and 0.001 ml respectively.*

illumination at the eyes was *E* meter-candles from a dazzle-source placed *D* degrees above the line of vision.

In making an observation the brightness of screen was adjusted to *F* ml, and the dazzle-source L_1 was placed *D* degrees above the line of vision and adjusted to give *E* mc at the observer's eyes, and then the brightness difference of test-object above its background was varied

until it could just be seen. The average minimum perceptible brightness differences, $(\Delta F)_r$, of the two observers for various values of F, E and D are given in Table 2. In Fig. 3 are plotted results for a constant angle D of five degrees for the dazzle-source, for various illuminations E at the eyes, and for various brightnesses F of the background. It may be observed that for a constant brightness F of background and for a constant angular position D of the dazzle-source, the minimum perceptible brightness difference $(\Delta F)_r$ increased in almost direct proportion to the illumination E at the observer's eyes from the dazzle-source.

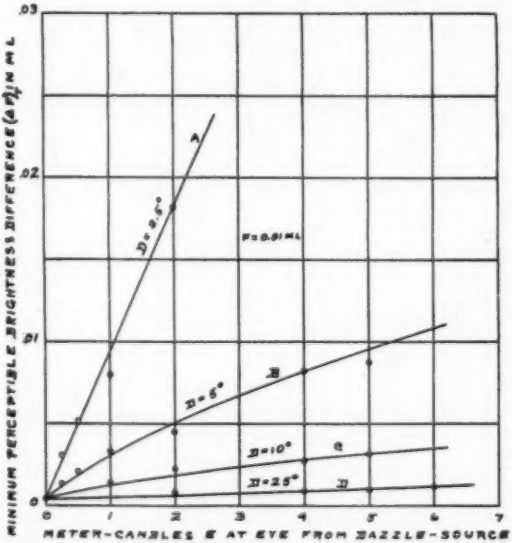

FIG. 4. *Curves A, B, C and D show the variation of minimum perceptible brightness difference* $(\Delta F)_r$ *with illumination E at eye from a dazzle-source located respectively 2.5, 5, 10 and 25 degrees above the line of vision when the background brightness F was 0.01 ml.*

Fig. 4 shows how for a constant brightness of background of 0.01 ml, the minimum perceptible brightness difference, $(\Delta F)_r$, varied with the illumination E at the eyes from the dazzle-source for various angles D of the dazzle-source above the line of vision. Curve A is for an angle D of 2.5 degrees; curve B for an angle of 5 degrees; curve C for an angle of 10 degrees; and curve D for an angle of 25 degrees. It should be noticed that for a given illumination E at the eyes from the dazzle-source, the minimum perceptible brightness difference $(\Delta F)_r$ required is greater the smaller the angle D.

In analyzing these results by the use of curve A, Fig. 2, it was found that the minimum perceptible brightness difference, $(\Delta F)_r$, required corresponded to an adaptive brightness greater than the actual background brightness F used. For a given background brightness F, and for a fixed position of the dazzle-source with reference to the test-object, the equivalent background brightness exceeded the actual background brightness by an amount approximately proportional to the illumina-

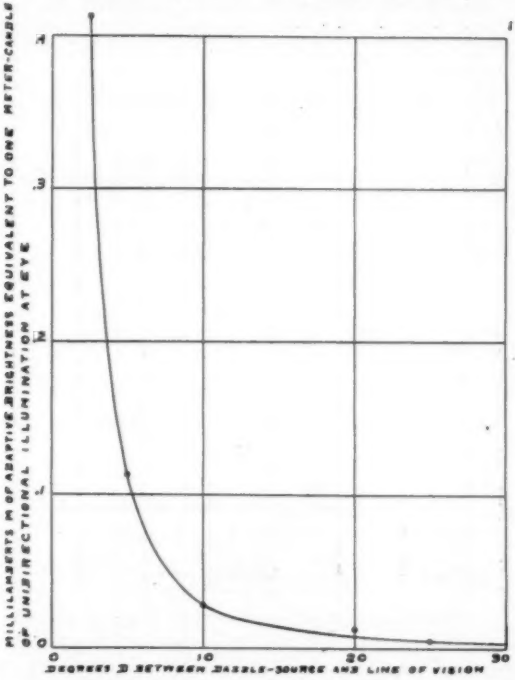

FIG. 5. *Shows the millilamberts increase of adaptive brightness equivalent to one meter-candle of unidirectional illumination E at eye from a dazzle-source located at various angles D to the line of vision.*

tion E at the eyes from the dazzle-source. Thus when the eye was exposed to the dazzle-source it appeared to function as though adapted not to a brightness of F but to a brightness of $F+ME$, where M is practically a constant for any given angular relation between dazzle-source and test-object. Results obtained by this method of procedure are given in column 5, Table 2. As shown by column 6 in the table or as may be inferred from the plot of results in Fig. 4, the values of M

are less the larger the angle of separation D of the dazzle-source and the line of vision. From Fig. 5 where values of M are plotted against angle D, it appears that average values of M are approximately satisfied by the empirical equation

$$M = 2.9/D^2 \qquad (1)$$

TABLE 2. *Table showing the minimum perceptible difference in brightness $(\Delta F)_T$ between test-object and background when the background had a brightness of F millilamberts and a unidirectional illumination of E meter-candles shone into the observer's eyes from a dazzle-source located D degrees above his line of vision.*

Brightness of background in ml F	Unidirectional illum. at eye in mc E	Degs. of dazzle-source above line of vision D	Minimum perceptible brightness Diff. in ml $(\Delta F)_T$	Adaptive brightness corresponding to $(\Delta F)_T$ or $F+ME$	Equiv. adaptive brightness per metercandle M in ml
1	1	5	.0225	1.122	.122
1	2	5	.0254	1.27	.135
1	4	5	.0312	1.622	.155
1	5	5	.0329	1.738	.148
				Average	.140
.1	1	5	.0050	.209	.109
.1	2	5	.0068	.299	.100
.1	4	5	.0090	.408	.077
.1	5	5	.0108	.525	.085
				Average	.093
.01	.25	5	.00141	.043	.130
.01	.5	5	.00208	.071	.122
.01	1.	5	.00338	.130	.12
.01	2.	5	.00452	.178	.084
.01	4.	5	.00822	.372	.091
.01	5.	5	.00879	.425	.083
				Average	.105
.001	.25	5	.00153	.0478	.187
.001	.5	5	.00174	.0562	.110
.001	1.	5	.00254	.0912	.090
.001	2.	5	.00550	.229	.114
.001	4.	5	.00844	.385	.096
.001	6.	5	.01160	.550	.092
				Average	.115
.00002	.5	5	.00178	.059	.118
Grand average for $D=5$ Degrees.					.113

TABLE 2. (Continued)

F	E	D	$(\Delta F)_T$	$F+ME$	M
.01	.25	2.5	.00312	.118	.430
.01	.5	2.5	.00521	.214	.408
.01	1.	2.5	.0080	.363	.353
.01	2.	2.5	.0183	.933	.462
Average for $D=2.5$ Degrees.					.413
.01	1	10	.00143	.0437	.0337
.01	2	10	.00220	.076	.0330
.01	4	10	.00277	.100	.0225
.01	5	10	.00321	.120	.0220
Average for $D=10$ Degrees.					.0278
.01	.25	20	.00059	.0141	.0164
.01	.5	20	.00070	.0182	.0164
.01	1.	20	.00084	.023	.0130
.01	2.	20	.00109	.0316	.0108
.01	4.	20	.00145	.045	.0088
Average for $D=20$ Degrees.					.0131
.01	2	25	.00082	.0213	.0056
.01	4	25	.00096	.0270	.0042
.01	5	25	.00096	.0275	.0035
.01	6	25	.00118	.0347	.0041
Average for $D=25$ Degrees.					.0043

Therefore, in discriminating brightness differences the observer's eyes appeared to function as though adapted to an

$$\text{equivalent total brightness} = F + 2.9E/D^2 \qquad (2)$$

millilamberts, where F was the actual background brightness in millilamberts, E the meter-candles of illumination at the observer's eyes from the dazzle-source, and D the elevation in degrees of the dazzle-source above his line of vision.

These results seem to show that the effect of a glare-source when located at an angle to the line of vision in the field of view is to increase the adaptive brightness of the fovea and thereby increase the minimum perceptible brightness difference (M.P.B.D.) required for the discrimination of objects.

After-Effects of Exposure to Glare-Source. In order to ascertain whether the change in adaptation of the fovea due to a glare-source in the field of view persisted after the glare-source was obscured, a

similar test to the preceding was conducted upon the same observers with the same arrangement of apparatus, only the glare-source was obscured an instant before an observation was made.

By analyzing the results in a manner similar to that employ d above, it was found that the after-effects upon brightness discrimination due to a previous exposure to a glare-source were of the same general character as its effects while the observer's eyes were still exposed to it, only their magnitudes were reduced. The magnitude of the after-effects varied somewhat with the length of exposure to the glare-source and with the lapse of time after it was obscured. It thus appeared that the after-effects were transient and subject to decay as after-images; or

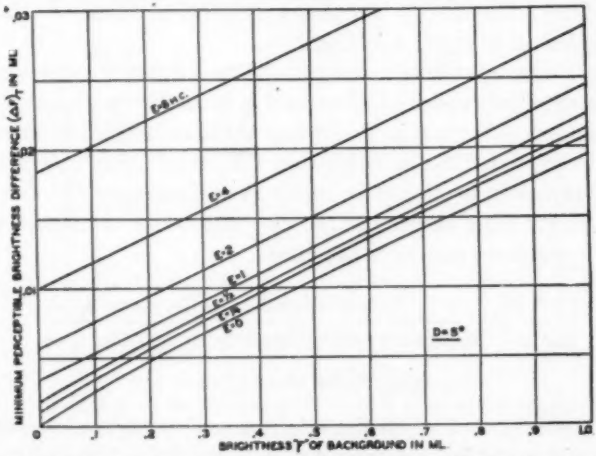

FIG. 6. *Showing how the minimum perceptible brightness difference $(\Delta F)_T$ varies with the background brightness F for unidirectional illuminations E at the eye of 0, 1/4, 1/2, 1, 2, 4 and 8 meter-candles from a glare-source 5 degrees out of the line of vision.*

subject to change, as adaptation, from a higher to a lower brightness. Observations made as soon as possible after the glare-source was obscured (probably not more than two seconds), showed that the effects of exposure to the glare-source had decreased to about one third their original value.

PART 2

Throughout the remainder of this article it will be shown how by a suitable application of the principles of glare just enunciated, it may be ascertained whether a given light-source aids vision more by illuminating an object than it hinders vision by glare. Although in experi-

mentally determining the effects upon vision of a glare-source in the field of view, such conditions prevailed as to render the object just visible, yet these laws may be assumed to hold true under conditions where the object is more or less clearly visible.

Variation of M.P.B.D. with Background Brightness for Various Illuminations E at Eye from the Glare-Source. In Fig. 6 are shown computed values of the minimum perceptible brightness difference $(\Delta F)_r$ for various background brightnesses F and for various values of the illumination E at the eye from a glare-source located at an angle D of five degrees out of the line of vision. The total equivalent adaptive brightness was assumed to be, according to Eq. (2), equal to $F+2.9E/D^2$ millilamberts and the corresponding values of M.P.B.D. $(\Delta F)_r$ scaled off of curve A in Fig. 2.

Safety-Factor of Visual-Contrasts. The brightness difference ΔF between an object and its background is generally much greater than the minimum difference in brightness $(\Delta F)_r$ at which the object can just be perceived. The ratio between the actual brightness difference and the threshold or M.P.B.D. serves as an insurance that the object be seen and is some measure of the ease with which the object may be seen. Therefore we may term the ratio,

$$\frac{\Delta F}{(\Delta F)_r} = \frac{\text{Actual brightness difference}}{\text{Minimum perceptible brightness difference}}$$

$$= \textit{a safety-factor of visual-contrasts} \qquad (3)$$

It is apparent that when the brightness of an object's background and the difference in brightness ΔF between the object and its background are kept constant the safety-factor of visual-contrasts increases as the minimum perceptible brightness difference $(\Delta F)_r$ decreases, and this latter decreases as the adaptive effect, KE/D^2, of the glare-source decreases. To illustrate this: assume, as shown at A in Fig. 7, that a 40 cp lamp is located at a distance of 2 meters from an observer's eyes and at an angle of D degrees to his line of vision. Assume also that the object has a reflection-factor $f=0.5$, and the screen has a reflection-factor $f_1=0.2$ and a brightness F of 0.1 ml. Then as the angle D of the glare-source with the line of vision is varied the safety-factor of visual-contrasts varies as shown in curve B, Fig. 7. It is apparent that the safety-factor becomes one at an angle D of about two degrees and at smaller angles the object cannot be seen.

Should the reflection-factor f of the object be adjusted to obtain a constant safety-factor of three, the requisite value of f for each angle D would be as shown in curve C, Fig. 7.

Variation of Safety-Factor of Visual-Contrasts with Illumination of Object and Background. If an observer views a screen upon which a flat object is placed, and a glare-source is in his field of view, the safety-factor of visual-contrasts may be increased by increasing the illumination upon the screen. This is illustrated in Fig. 8. With a given brightness F of the screen, the greater the illumination E at the eye from the glare-source the lower the safety-factor.

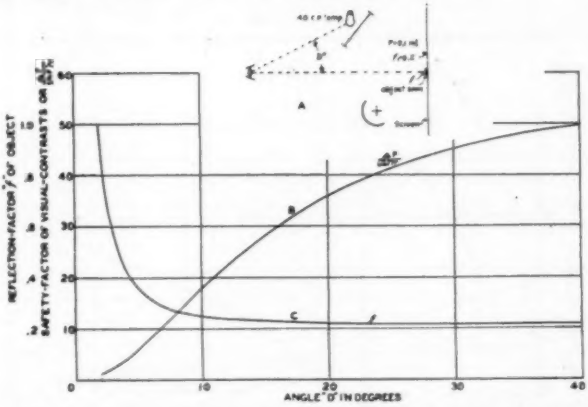

FIG. 7. *Sketch A shows conditions assumed. Curve B shows how the safety-factor of visual-contrasts varies as angle D of glare-source varies when the reflection-factor of object is 0.5. Curve C shows how the reflection-factor of the object would vary if the safety-factor were to be maintained at a constant value of three.*

Conditions Under Which a Light-Source May or May Not Assist Vision. When an object and its background are illuminated by a common source of light out of the field of view, vision is in general improved by an increase of its illumination until brightnesses of several hundred millilamberts are attained. It has been shown that when a light-source is placed in the field of view but does not illuminate the object or its background, vision is in general impaired. If then a light-source illuminates an observer's eyes as well as the object to be seen and its background, the question arises as to whether a given light-source is advantageous, and if so, to what extent.

At G in Fig. 9 an eye is represented viewing an object with a reflection-factor f of 1/3 placed against a background with a reflection-

factor f_1 of 1/2. The light-source L_1 is placed at an angle of 5 degrees to the observer's line of vision and is so arranged that it gives an equal illumination E in a plane at right angles to its propagation at the observer's eye, and upon the screen. In addition the object and background receive sufficient illumination from the light-source L_2 to give

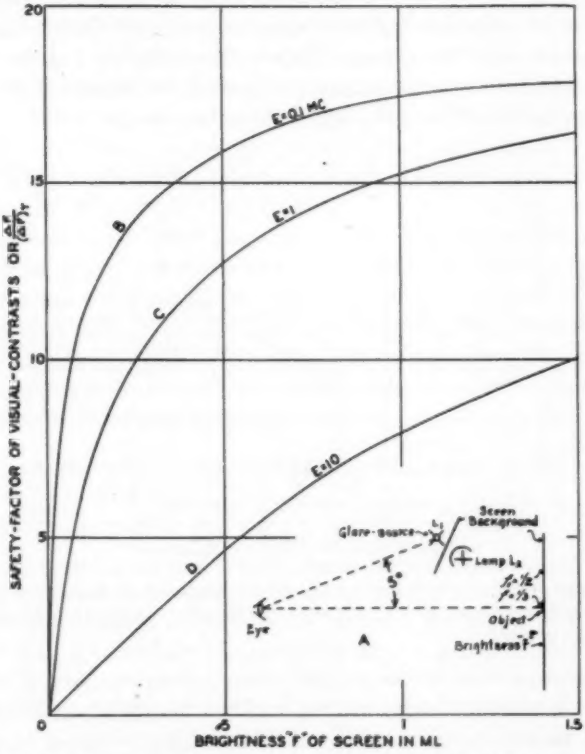

FIG. 8. *Sketch A shows conditions assumed. Curves B, C and D show how the safety-factor of visual-contrasts varies with the background brightness F for values of the illumination E at the eye from the glare-source of 0.1, 1 and 10 mc respectively.*

the background an initial brightness of F_0 millilamberts. Curves A, B, C, D, E and F in Fig. 9 show the safety-factor of visual-contrasts for various values of E when the initial brightness F_0 of the background is 0, 0.001, 0.01, 0.1, 1, and 10 ml respectively. It may be observed that curve A (for $F_0=0$) shows a continuous increase of safety-factor with increasing values of E and that curves B and C show minima

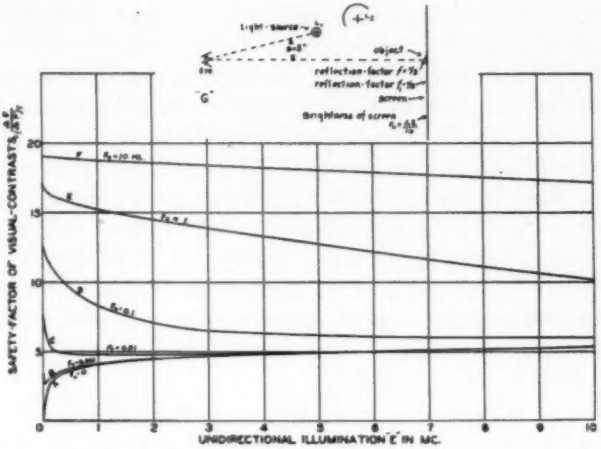

FIG. 9. *Sketch G shows conditions assumed. For equal illuminations at the eye and upon the screen from the light-source L_1, the curves show how the safety-factor of visual-contrasts varies with the illumination E at the eye and screen from the light-source L_1. Curves A, B, C, D, E and F are for initial brightnesses of the screen of 0.0, 0.001, 0.01, 0.1, 1 and 10 ml respectively.*

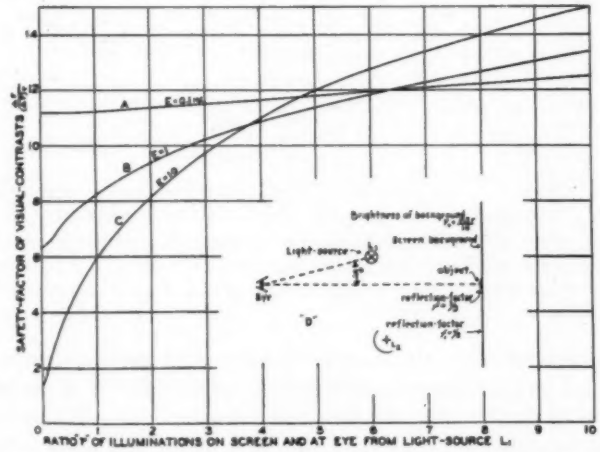

FIG. 10. *Sketch D shows the light-source L_1 to be 5 degrees out of the line of vision and to give illuminations of E at the observer's eye and rE upon the screen. The illumination from light-source L_2 upon the screen produces an initial brightness F_0 of 0.1 ml. Curves A, B and C show how the safety-factor of visual-contrasts varies with the ratio r for illuminations E of 0.1, 1, and 10 mc, respectively.*

at values of E of 0.03 and 1 mc respectively. Curves D, E and F show light-source L_1 to be a disadvantage.

In Fig. 10 are shown values of the safety-factor of visual-contrasts computed for an initial background brightness F_0 of 0.1 ml and for an illumination upon the background from light-source L_1 of rE meter-candles, where r may have any fractional or multiple value of one, and E is the illumination at the eye from the light-source L_1 placed at 5 degrees to the line of vision. The advantages of small values of E and large values of r are clearly shown.

Curves A, B, C, D and E in Fig. 11 show what initial brightness F_0 of screen is necessary for a safety-factor of visual-contrasts of ten when

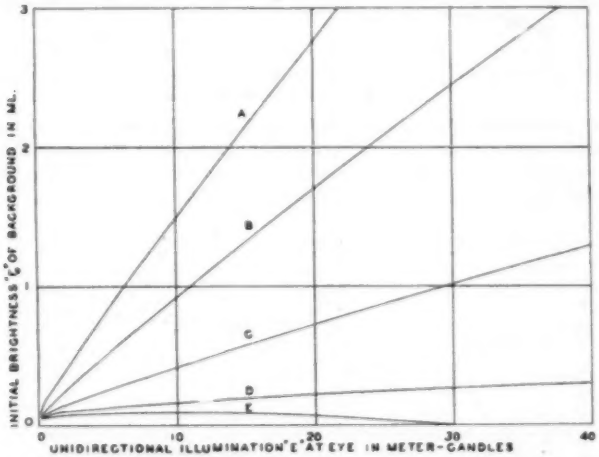

FIG. 11. *Shows the initial brightness F_0 of the background in order that the safety-factor of visual-contrasts may be ten when the conditions are as shown in Sketch D, Fig. 10. Curves A, B, C, D and E are for ratios r of the illumination upon the screen to that at the eye of 0, 1, 2, 2.5 and 2.7, respectively.*

the illumination upon the screen from the glare-source is respectively 0, 1, 2, 2.5 and 2.7 times its illumination at the eye. It is assumed, as shown in Sketch D, Fig. 10, that the reflection-factor of the screen is 1/2, the reflection-factor of the object is 1/3, and the angle D between the glare-source and line of vision is 5 degrees. It is evident, under the conditions assumed, that when the initial brightness of the screen is about 0.07 ml, and the illumination on the screen from the glare-source is about 2.7 times its illumination at the eye, the obscuring effect of the glare-source is just compensated.

Under similar conditions to the preceding, it would be interesting to know what the illumination upon the screen from the glare-source in terms of its illumination at the eye should be in order that the obscuring effect of the glare-source may be compensated by its illumination upon the screen for the glare-source at various angles D to the line of vision. With an initial brightness of the screen of 0.1 ml, a reflection-factor of the screen of 1/2, a reflection-factor of the object of 1/3, and an illumination E at the eye from the glare-source of 10 mc, then for a safety-factor of visual contrasts of 10, the ratio r for various angles D of the glare-source are as follows:

Angle of glare-source to line of vision, D in degrees	Rate of illumination upon the screen from the glare-source to its illumination at the eye, r
5	2.6
10	0.7
20	0.14
30	0.02

LIGHTING RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,
NELA PARK, CLEVELAND, OHIO.

Magnetic properties of single crystals of iron. One of the crystals used in these experiments was 68x2.4x1.8 mm, five others were 40x2x2—much the largest thus far employed in research of this type. The first-named crystal rod (its axis inclined at 21°10′ and 70°10′ to the tetragonal axis) was used for determining an *I-H* curve which is of most curious shape, being composed of three nearly straight lines the last of which is parallel to the *H*-axis at $I=1710$; it commences at $H=330$ cm, so that saturation is much more easily attained than in the case of polycrystalline material. The author next measured hysteresis loss in terms of the reciprocal area of the single crystal (?) and found it increasing very rapidly, the curve again consisting of two straight lines. The magnetostriction of the set of five samples above mentioned, the axes of which were differently inclined to the crystal axes, depends obviously very much on this inclination; in one case it was a continual increase approaching saturation, in the other four an increase followed sooner or later by a decrease; the authors think that the magnetostriction is positive along the tetragonal and negative along the trigonal axis.—[K. Honda *et al.* (Tokyo); Nature, *117*, pp. 753–754; 1926.]

KARL K. DARROW

Atom-lattice of Heusler alloys. The atom-lattices of 12 ferromagnetic ternary alloys of Al, Cu, Mn with varying proportions of the three metals were determined by the Debye-Scherrer method. Each displayed the diffraction-pattern of a body-centered cubic lattice of spacing 2.975A, and most displayed in addition the characteristic diffraction-patterns of a body-centered cubic lattice of spacing 2.903A and a face-centered cubic lattice of spacing varying from sample to sample and decreasing as the proportion of Al is increased. The relative intensities of the lines of each pattern indicate that each lattice is homogeneous; so that each lattice must consist exclusively of light (Al) or exclusively of heavy (Cu or Mn) atoms, or else if any lattice contains atoms of both kinds they must be distributed at random over the lattice points. A single ternary alloy which was not ferromagnetic also failed to show the first-named lattice. Assuming that this first lattice consists of Cu and Mn atoms alternately, the distance from an atom of either kind to its neighbor of the other kind agrees very closely with half the sum of the atom-to-atom distances in pure Cu and pure Mn, which is suggestive. No such agreement could be found for the other lattices.—[L. Harang (Oslo); Phys. ZS., *27*, pp. 204–205; 1926.]

KARL K. DARROW

SPECTRA EXCITED BY ACTIVE NITROGEN*

By Arthur Edward Ruark, Paul D. Foote, Philip Rudnick, and Roy L. Chenault

I. INTRODUCTION

E. P. Lewis[1] discovered in 1904 that the spectra of gases or vapors can be excited by allowing them to mix with active nitrogen. Since that time the phenomenon has been studied by Strutt and Fowler,[2] Mulliken,[3] Worthing and Rudy,[4] and others. The spectra of monatomic metallic vapors excited by this method are especially interesting. In general, only the lines of the neutral atom are present, and in some cases only a portion of the arc spectrum has been observed.

We shall review briefly the evidence in favor of the view that active nitrogen consists of neutral unexcited nitrogen atoms, and shall describe new experiments supporting this hypothesis. From data on the Hg spectrum a lower limit is obtained for the energy which can be transferred to an atom in the nitrogen afterglow. It is shown that intensity relations for Hg spectra excited in the afterglow are different from those characteristic of spectra produced by other methods. Data on the excitation of other metals by active nitrogen are given, and thermochemical evidence bearing on the mechanism of excitation is introduced.

In describing spectral terms, we use the numeration of Paschen and a notation in which superscripts indicate term multiplicity, and subscripts represent inner quantum numbers. For example, the line 2536A of Hg will be written $1^1S_0-2^3P_1$.

II. THE NATURE OF ACTIVE NITROGEN

The preponderance of evidence is to the effect that active nitrogen consists of neutral nitrogen atoms in an unexcited state. This hypothesis explains qualitatively all the effects produced by active nitrogen with which we are acquainted. However, many phenomena associated with active nitrogen can be also explained on the hypothesis that it

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[1]Astrophysical Journal, *20*, p. 49; 1904.

[2]Proc. Roy. Soc., *85*, p. 377, 1911, and *86*, p. 105; 1911.

[3]Phys. Rev., *23*, p. 767, 1924; *25*, p. 259, and *26*, p. 1; 1925.

[4]Phys. Rev., *23*, p. 767; 1924.

consists of metastable neutral nitrogen molecules. Saha and Sur[5] discussed the data from this point of view, and some of the authors[6] were at one time of this opinion. The hypothesis of neutral nitrogen atoms was discussed convincingly by Miss Sponer[7] who studied carefully the spectra of the low-voltage arc in nitrogen. She assumes that in triple collisions of two nitrogen atoms with a metal atom, the heat of formation of a nitrogen molecule can be utilized to excite the metal atom. Similarly, when two N-atoms encounter an N_2-molecule, they may combine and the heat of association will raise the N_2-molecule to an excited state. (In general, it is impossible for two nitrogen atoms alone to combine in collision, because usually the laws of conservation of energy and momentum cannot be satisfied in such a process.[8]

The assumption of such triple collisions is in agreement with the measurements of Angerer[9] and Rudy[10] on the rate of decay of the afterglow. Both these investigators found a brightness-time curve which could be explained on the hypothesis that two activated entities are involved in each elementary process of light emission.

Indirect evidence in favor of the atomic nitrogen hypothesis can be obtained by exhibiting the defects of rival theories. If active nitrogen were a metastable[11] form of the molecule, the high energy of activation suggests the possibility of the presence of a band absorption lying in the neighborhood of the visible spectrum. We have made experiments to test this prediction. Active nitrogen produced in the electrodeless discharge, at a concentration of possibly one to two per cent, was pumped through a tube 2 cm in diameter and 4 meters long at a nitrogen pressure of about 0.15 mm. Light from an under-water spark, giving a continuous spectrum, was passed through

[5]Phil. Mag., *48*, p. 421; 1924.

[6]Foote, Ruark, and Chenault, Phys. Rev., *25*, p. 241; 1925, and Nature, *114*, p. 750; 1924.

[7]ZS. f. Physik, *34*, p. 622; 1925. See also Duffieux, Nature, *117*, p. 302; 1926; Duncan, Astrophysical Journal, *62*, p. 145; 1925; and Kneser, Ann. Physik, *79*, pp. 585 and 597; 1926. The potentials of Duncan, as corrected by Birge, Nature, *117* p. 81; 1926, are in close agreement with those of Sponer. Kneser finds a high potential (14.8) for the excitation of the second positive group than Sponer and Duncan, who obtain 13.0 and 12.0 volts respectively; in other respects his values agree well with theirs. His hypothesis that the first positive group is due to negatively charged molecules cannot be correct in view of Strutt's work on the electrical properties of active nitrogen, reviewed in Section III of this paper.

[8]Born and Franck, ZS. f. Physik, *31*, p. 411; 1925.

[9]Phys. Zeit., *22*, p. 97; 1921.

[10]Phys. Rev., *27*, p. 110; 1926.

[11]Birge, Nature, *114*, p. 642; 1924.

the tube. The spectrum of the transmitted light was examined between the limits 6500A and 3200A, and no trace of absorption was found. In so far as this null result can yield any information, the observation is consistent with the atomic nitrogen hypothesis, for spectra obtained by Hopfield with the vacuum spectroscope show that the absorption lines of the neutral atom lie in the extreme ultraviolet.

Evidence that the entities which cause the afterglow are not charged may be drawn from Strutt's observation that a stream of the glowing nitrogen cannot be deflected by an electric field. He also discovered that no appreciable current flows between sounding electrodes which are not in actual contact with the glowing gas. This could not be the case if all the entities which give rise to the characteristic effects of active nitrogen were charged, for chemical experiments by Strutt[12] show that frequently one to two per cent of the gas is activated.[13] However, saturation currents of the order of 10^{-5} amperes were obtained when the electrodes were located within the afterglow. These currents are probably due to the liberation of electrons at the electrode surface by active nitrogen.[14]

It will be understood, of course, that secondary effects, due to excited molecules of nitrogen or other substances, may become conspicuous at higher pressures. For this reason most of our work has been done in the electrodeless discharge which permits production of the afterglow at pressures 10 to 30 times smaller than those which give the best results in the condensed discharge. For example, at pressures of .01 to .02 mm we have obtained fully exposed mercury spectra which show only a faint trace of the nitrogen afterglow bands.

It is not to be supposed that the primary effect always consists of a triple collision resulting in the production of an unexcited N_2-molecule and an excited atom or molecule. There are many examples of more complex processes. For instance, Strutt and Fowler found that the vapor of $SnCl_4$ yields the raies ultimes of tin, and that HgI_2

[12]Proc. Roy. Soc., *86*, p. 56; 1911.

[13]Detailed evidence on the conductivity of active nitrogen has been accumulated in recent years by S. Karrer, C. S. Fazel, and B. V. Cassen at the Fixed Nitrogen Research Laboratory. See abstract, Phys. Rev., *23*, p. 297; 1924.

[14]The possibility that an excited atom can liberate an electron from a metallic surface on which it impinges, in case the energy of the atom is greater than the work function of the surface, was suggested by Holst and Oosterhuis (Phil. Mag., *45*, p. 1117; 1923) on the basis of experiments made with neon. It is supported by Webb's work on the diffusion of excited mercury atoms. (Phys. Rev., *24*, p. 113; 1924.)

gives bands characteristic of the compound together with 2536A of Hg, and an I_2 band at about 3430A.

III. EXPERIMENTAL ARRANGEMENTS

Many types of discharge tube were utilized in this investigation. The one which was found most convenient for work at low pressures was a spherical bulb about 30 cm in diameter, in which an electrodeless discharge was produced by means of a few turns of heavy wire wound around the bulb. (See Fig. 1). A sectored disc, carrying a commutator on its axle, is placed in front of the spectrograph slit. The commutator closes the circuit for exciting the discharge just

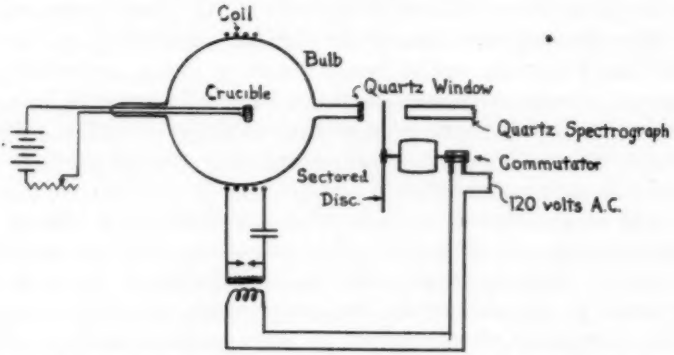


FIG. 1. *Electrodeless discharge bulb and accessory apparatus for photographing spectra excited in the afterglow of active nitrogen.*

after the sectored disc covers the slit of the spectrograph, and opens it just before the slit is uncovered, so that only the light of the afterglow can reach the spectrograph. Depending on the conditions of the experiment, the afterglow persists from one-half second to five minutes. It was seldom found necessary to run the sectored disc faster than one revolution per second. At the center of the discharge bulb there is a small crucible surrounded by a heater of nichrome wire which contained the metal, the spectrum of which was to be studied. Experiments were also carried out with the metal resting on the bottom of the bulb. In this case its surface was vaporized by ionic bombardment, since it lay directly in the brilliant discharge which occurs in the plane of the coil. However, the vapor density along the line of

sight was too small to yield the spectrum of the metal. With thallium, after the discharge was cut off a green glow was obtained extending about 5 cm from the lump of metal. With a different design of the bulb it would have been possible to obtain the spectrum of the glow very easily. This method of getting the metal into vapor form may prove very useful in studying the spectra of metals of high boiling point, excited by active nitrogen.[15]

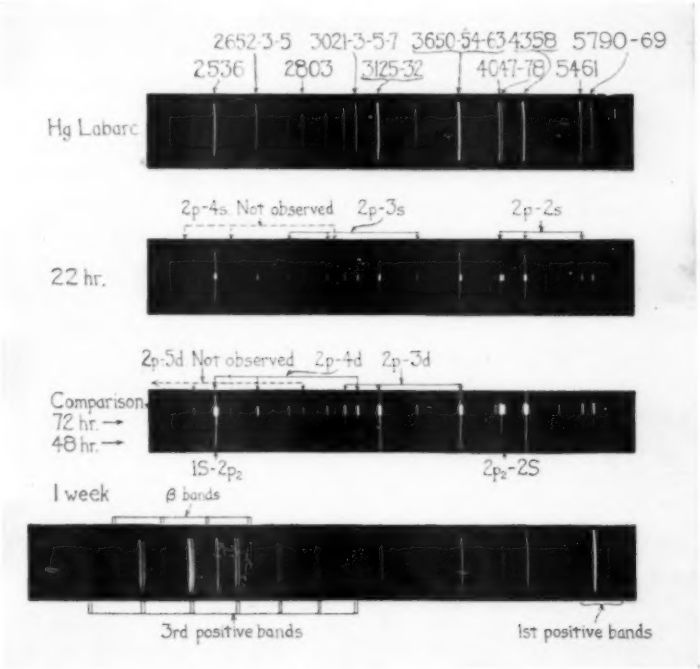


FIG. 2. *Spectra of Hg excited by active nitrogen, showing also the afterglow bands of* N_2.

Metallic nitrides accumulate in the apparatus in considerable amounts. Nearly all of these nitrides are explosive, and some are dangerously so. In the case of some metals, such as mercury, the nitrides are deposited on the walls during the period of afterglow, but are broken down by the succeeding discharge so that the nitrogen

[15]Robertson (Phil. Mag., *50*, p. 752; 1926) has made similar observations and has obtained very interesting spectra of cold metals placed in an electrodeless discharge bulb. Generally these spectra show only the resonance lines of the arc and spark spectra of the metals.

pressure does not diminish, and the apparatus can be sealed off from the nitrogen supply if desired. In other cases the nitride is more stable and is formed so readily that the afterglow is completely destroyed.

A Hilger quartz spectrograph (Model E2) was used. The exposures were generally 24 hours long, although in special cases they ran for a week.

IV. THE BAND SPECTRUM OF THE AFTERGLOW

The spectra of Fig. 2 show that by proper choice of the nitrogen pressure and the vapor pressure of the metal to be studied it is possible either to subordinate band spectra or to make them quite bright relative to the spectrum of the metal. The bands appearing on our one week exposure are practically identical with those recorded by Strutt and Fowler. Especially prominent are the "β bands" and the "3rd positive bands," or γ bands of Strutt and Fowler. Birge[11] and Mulliken[3] have concluded that these bands are due to NO, and this is supported by a large amount of spectroscopic evidence.[16] We have recorded two new bands of the system called β, lying near 2400 and 2300 A, respectively. Fowler and Strutt[2] observed the stronger members of the second positive group of N_2 but thought they might be due to reflected light from the discharge tube or to stray discharge, since Lewis did not obtain them when using a sectored disc and an intermittent discharge. Later Strutt[17] made a special examination of this point and concluded that they were not produced in carefully purified nitrogen which contained, however, a small amount of mercury vapor from the pumps. We find that the stronger bands of the group are well developed under the following conditions: Nitrogen pressure, 0.2 to 0.3 mm; mercury vapor pressure, 0.02 mm; small amounts of oxygen present (proved by the strong development of the β and γ bands due to NO); exposure, 16 hours to 33 hours. The following bands were identified and others may be present: 3577, 3536, 3371, and 3159A. This observation is not to be looked upon as conflicting with that of Strutt because the physical conditions of the two experiments are radically different. We have also obtained these bands at a nitrogen pressure of 0.01 to 0.02 mm and a mercury vapor pressure of .001 mm. Their intensity relative to neighboring arc lines of mercury is not markedly different from that observed at

[16] See Sponer, Nature, *117*, p. 81; 1926.
[17] Proc. Roy. Soc., *A*, *93*, p. 254; 1917.

higher pressures. According to Miss Sponer (loc.cit.) the excitation potential for the 0→0 band at 3371A is 13.0 volts. This is higher than her estimate of the heat of association of nitrogen, 11.4 volts.[18] If we accept the latter value, these bands must be due to some sort of successive excitation. Our experimental evidence does not suffice to decide the question.

V. THE AFTERGLOW SPECTRUM OF MERCURY

We have obtained mercury spectra under a great variety of conditions, using the apparatus of Fig. 1. Some plates were exposed very heavily to obtain the higher members of the triplet subordinate series. On a plate exposed for 60 hours with 0.02 mm nitrogen pressure, and with the mercury at room temperature, the sharp series was recorded to 2^3P-4^3S and the diffuse series to $2^3P_1-6^3D$. The excitation potential of 6^3D is 10.0 volts. This is a lower limit for the heat of association of nitrogen, on the hypothesis that active nitrogen is atomic nitrogen.

It was impossible to determine whether the lines 5790 and 5769A were present because they are covered by the yellow afterglow bands. The lines observed on our plates are as follows:

2^3P-m^3S: 5461, 4358, 4047; 3343, 2893, 2752; 2925, 2576, 2464.
2^3P-m^3D: 3662, 3654, 3650, 3131.6, 3125, 2967.2;
3025, 3023, 3021, 2653 , 2652, 2534 ;
2805, 2803, 2802, —, 2482, —;
2699, —, —, —, —, —;
$2^3P_1-2^1S_0$: 4078,
2^3P-4^1D: 3663, 3131.8, 2967.5, 3027.5, 2655,
$2^1P-m^1S_0$: 4916, 4108.
$2^1P-m^1D_2$: 4347,
$1^1S_0-2^3P_1$: 2536.

Some of the spectrograms are reproduced in Fig. 2. The short lines in the second and third spectra are comparisons. The fourth spectrum shows the first positive bands of nitrogen and the "third positive bands" and "β bands" of NO. All other lines in these spectrograms are due to mercury.

A separate series of plates was taken for intensity measurements. Photographic density measurements were made and the results were compared with those obtained from the spectra of the electrodeless discharge in N_2-Hg and H_2-Hg mixtures, the labarc, and low-current ,

[18] Phys. Rev., *27*, p. 641; 1926.

low-voltage arcs in Hg at 25 volts and 16 volts. It was found that the rate of decrease of intensity in passing to higher members of the triplet subordinate series is much greater for excitation by active nitrogen than in any of the sources mentioned. The effect is more striking for the diffuse series than for the sharp series. Furthermore, the ratio of the total intensity of the diffuse series to that of the sharp series is smaller in the afterglow spectrum than in any of the other sources mentioned. Certain lines are much enhanced as compared with their intensity in the comparison sources, while others are considerably weakened.

The results for the 3650 group are as follows:

λ in A	Classification	Intensity
3663	2^3P_2-3^1D_2	Weakened
3662	2^3P_2-3^3D_2	"
3654	2^3P_1 3^3D_2	"
3650	2^3P_2-3^1D_1	Enhanced

The results for the 3125 group, $2^3P_1-3^1D$ and $2^3P_1-3^3D$, are similar. The meaning of these observations is that in excitation to ^3D orbits by triple collisions, the $^3D_1$ orbit is favored at the expense of the others. This is in close analogy with Rudy's[19] statement that band spectra excited by active nitrogen show low rotational energy.

VI. SPECTRA OF OTHER ELEMENTS IN ACTIVE NITROGEN

In this section we shall discuss additional spectral data which have a definite significance in determining the energy of active nitrogen.

In the spectrum of zinc excited by active nitrogen, Lewis found the first two members of each triplet subordinate series. The highest orbit involved in the production of these lines has an excitation potential of 8.46 volts, while the ionization potential of Zn (9.35 volts) is considerably less than the energy which can be transferred to mercury atoms by active nitrogen. Saha and Sur[5] pointed out that it should be capable of ionizing Mg and exciting the resonance lines of the spark spectrum. These lines were not observed by Strutt and Fowler, but they state that the spectrum obtained was not very strong. This matter requires further examination. Though these ionization processes are energetically possible, we have no way of predicting whether they actually occur; each case must be examined experimentally.

[19]Phys. Rev., *27*, p. 110; 1926.

We have as yet no definite evidence that metal atoms can be ionized by active nitrogen. On the other hand, we know definitely that two electrons may be displaced to higher orbits simultaneously, for Strutt and Fowler observed $2^3P_1-2^3P_1'$ of magnesium at 2779A,[20] in the afterglow. It is hard to understand why the lines 2^3P-4^3S did not appear, since their excitation potential is less than that of $2^3P_1-2^3P_1'$. At first sight it seems remarkable that only one line of the 2^3P–$2^3P'$ group was observed, but possibly the explanation is that the line recorded is the strongest member of the group.

Strutt and Fowler state that thallium gives numerous sharply defined lines belonging to the arc spectrum. We have obtained the following:(The intensity of 3519A is put equal to 10).

I. A.	Classification	Intensity	
		Fowler's Report	Authors
5350.46	2^3P-2^3S	10	
3775.72		10	6
3229.75	2^3P-3^3S	10	2
2580.14		8	1
2826.16		8	0
3529.43	2^3P-3^3D	8	4
3519.24		10	10
2767.87		10	3
2921.52	2^3P-4^3D	6	00?
2918.32		10	3
2379.58		8	Absent
2710.67 2709.23	2^3P-5^3D	4+8	1
2609.77 2608.99	2^3P-6^3D	4+6	0

Platinum lines excited by the afterglow were obtained by Lewis,[21] using a condensed discharge in a tube with platinum electrodes. Most of the lines recorded arise from transitions to low-lying levels of the neutral atom, as can be seen from the classifications given by McLennan and McLay,[22] and by Meggers and Laporte.[23]

[20]See Ruark, J. O. S. A. & R. S. I., *11*, p. 199; 1925.
[21]Astrophysical Journal, *20*, p. 49; 1904.
[22]Trans. Royal Society of Canada, *20*, p. 201; 1926.
[23]Phys. Rev., *28*, 642; 1926.

We owe to Jevons[24] a study of the spectra developed by admitting silicon and titanium tetrachlorides into active nitrogen. He obtained a large number of silicon lines, all of which are attributed by Fowler[25] to neutral silicon. Most of the lines found by Jevons have been classified by Fowler. Those which remain unclassified are 3905.51, 3020.01, 2987.65, 2631.30, 2568.66, and 2532.39A. Jevons has found that the afterglow gives a well-developed spectrum of titanium when titanium tetrachloride is introduced. This element has a low ionization potential, probably in the neighborhood of 6.5 volts, and when one electron has been removed the stronger spark lines are easily excited.[26] Jevons found a few of the lines of singly ionized titanium in the afterglow, but unfortunately does not give their wave lengths. The conditions under which these lines were produced are complicated, so that this observation does not tell us whether the titanium atom can be ionized and the ion excited in one triple collision.

Sodium and cadmium appear to destroy the afterglow. We were unable to photograph any sodium lines, although both metallic sodium and NaCl were tried out in the crucible of Fig. 1. With cadmium, long exposures yielded only the resonance line 3261A.

VII. THERMOCHEMICAL EVIDENCE ON THE ORIGIN OF SPECTRA EXCITED BY ACTIVE NITROGEN

It remains to examine whether metallic spectra produced by active nitrogen result from the production of metallic nitrides as the present Lord Rayleigh suggested in 1911. All metallic nitrides of the type RN_3 are highly endothermic, so we conclude that the excitation of the metal atom must occur in secondary processes, following the formation of the nitrides, if we accept at all the theory that they are essential to the production of these spectra. Let us see whether these spectra can be produced by the dissociation of nitrides in the afterglow. Berthelot and Vielle[27] measured the heat of explosion of HgN_3. They found

$$Hg+\tfrac{3}{2}(N_2)\rightarrow HgN_3-71.4 \text{ kg cal/mol.}$$

The heat of vaporization of Hg is 13 kilogram calories. If S is the unknown heat of sublimation of HgN_3, we see that the decomposition of HgN_3 in the gaseous state will yield $71+S-13$ kilogram calories

[24]Proc. Roy. Soc., *A*, *89*, 187; 1913.

[25]Philosophical Transactions, *A*, *225*, p. 1; 1924.

[26]See H. N. Russell's summary of the more important multiplets of astrophysical interest. Astrophys. Journal, *61*, p. 223; 1925.

[27]Ann. Chem. Phys. (7), *2*, p. 351; 1894.

per mol. A glance at any compilation of thermochemical data will convince the reader that S is certainly not larger than 50 kilogram calories and is probably much smaller. Assuming that it is smaller, the heat of decomposition of HgN_3 must be less than 108 kilogram calories, corresponding to 4.7 volts. We conclude that the afterglow spectrum of Hg is not produced *directly* by the decomposition of HgN_3. Of course secondary processes initiated by the decomposition may result in activated atoms and molecules of considerable energy. For this reason it is unsafe to draw conclusions about the mechanism of light production from thermochemical data alone. Considerations of this kind have been discussed by O. R. Wulf[28] in a paper on the spectra emitted by decomposing ozone.

VIII. SUMMARY

(1) No absorption spectrum of active nitrogen was found between 6500A and 3200A. This is in agreement with the hypothesis that active nitrogen consists of unexcited atoms, the absorption spectrum of which should lie in the far ultraviolet.

(2) New bands of the β system of NO are recorded.

(3) The second positive bands of nitrogen appear in the afterglow spectrum at 0.02 mm nitrogen pressure, and also at higher pressures when mercury is present.

(4) Mercury lines are not readily, if at all, excited by oxygen which has been exposed to the electrodeless discharge.

(5) Mercury atoms can receive at least 10 volts energy from active nitrogen. This is a lower limit for the heat of association of nitrogen if we admit that active nitrogen is atomic.

(6) The rate of decrease with increasing term number of the intensity of the mercury triplet subordinate series is very great in afterglow spectra.

(7) The usual rules for multiplet intensities are not valid for afterglow spectra.

(8) The afterglow spectra of Zn, Mg, Pt, $SiCl_4$, $TiCl_4$, Na, Cd, and Tl are discussed.

(9) It is pointed out that thermochemical data alone can give only negative evidence as to the origin of these spectra.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
JUNE 28, 1926.

[28] Proc. Nat. Acad. Sci., *12*, p. 129; 1926.

Absorption and resonance-radiation in excited helium. This is an experiment designed to show that when a gas is so excited that some of its atoms enter into states other than the normal, it becomes able to absorb lines of its own spectrum which are emitted in transitions into those states from others of higher energy. The noble gases lend themselves well to experiments of this sort since their spectra contain in the visible region many lines of this character, but none which are emitted in transitions into the normal state, so that the undisturbed gases present no absorption-lines in the visible. The gas in these experiments is contained in a wide tube ending in a capillary; the gas in the capillary is stimulated by alternating electric field violently enough to be luminous, and its image is projected through the gas in the wide tube, similarly but less violently excited, upon a photographic plate; on this the lines appear reversed. It is curious that the reversal is perceived with lines due (in emission) to transitions out of the D and d states into the $2P$ and $2p$ states respectively, but not with lines due to transitions out of the S and s states into these same two final states. Lines due to transitions into $2S$ and $2s$ from the P and p states respectively are reversed; and there is resonance of the line $2s-3p$, detected by a photographic plate exposed to the weakly excited gas but screened from the capillary. (During the present confusion in notation, especially in assignments of total quantum-number, it would be well to give diagrams of the stationary states). Compare the experiment of K. W. Meissner (This *Journal*, *11*, p. 146; *12*, p. 651).—[W. H. McCurdy (Johns Hopkins); Phil. Mag., (7) *2*, pp. 529–538; 1926.]

KARL K. DARROW.

Polarization of atom-nuclei invoked to interpret alpha-ray scattering. It is well known that recent experiments in Rutherford's laboratory show departures of the distribution-in-angle function of scattered alpha-particles from the form entailed by inverse-square force acting between the alpha-particles and nuclei; these departures occurr when alpha particles penetrate closely to nuclei of Al and Mg. Hardmeier and Debye have developed the consequences of the idea that the alpha-particle polarizes the nucleus, which presents an electric moment parallel to the radius-vector of the alpha-particle swinging around it and proportional to the strength of the field produced at the nucleus by the alpha-particle. The factor of proportionality between field and moment is the only disposable constant of the theory; yet with it Hardmeier produces what seems to be a remarkably good representation of the data.—[W. Hardmeier (Technische Hochschule, Zürich); Phys. ZS., *27*, pp. 574–576; 1926.]

KARL K. DARROW.

EXTINCTION COEFFICIENT MEASUREMENTS WITH DIVERGING LIGHT

By Donald Statler Villars[1]

In photochemical work it is desirable to have a quantitative knowledge of the extinction coefficients of the substances which absorb the light. When monochromatic light is used it would be convenient if one were able to measure these coefficients with the same apparatus which is to be used for the photochemical reaction. Ordinarily the extinction coefficient is calculated by substituting in Eq. (9) below, the ratio of observed light intensities transmitted by solvent and solution. Most investigators have felt it necessary to use parallel light, as otherwise d would not be constant. This has been produced at the expense of the intensity by cutting down the source to a point and using suitable lenses. When a monochromator is used this is not necessary because the light beam coming from the lens through the slit does not diverge at an angle large enough to cause an error as great as the experimental error due to intensity variation in the lamp. The purpose of this article is to demonstrate this mathematically and to give an illustration of the method of measurement recommended.

Let L = total light of monochromator slit transmitted by solution per unit time

$L_s = dL/ds$ = light transmitted per unit time per unit slit length

$L_{s\Omega} = d^2L/ds\, d\Omega$ = light transmitted per slit length per solid angle per unit time

where $d\Omega = da/r^2$ = small solid angle.

Then

$L_{s\Omega} ds d\Omega$ = light energy in solid angle $d\Omega$ from slit length ds

If Beer's law holds, the decrease in light energy will be proportional to the energy, the concentration, and the distance through which the light travels.

$$-d(L_{s\Omega} ds d\Omega) = k(L_{s\Omega} ds d\Omega) c dr$$

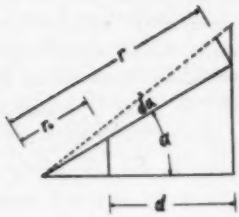
and

$$L_{s\Omega} = L_{0s\Omega} e^{-kc(r-r_0)} \qquad (1)$$

where the subscript $_0$ refers to the light rate at the front cell wall.

[1]National Research Fellow in Chemistry.

The light from the circular lens passing through the element ds of the slit will diverge in the shape of a cone with apex at the slit. If we now integrate the light transmitted by concentric rings of the circular cross section of the beam at the rear cell wall, we will obtain the total transmitted light from the point ds on the slit.



$$L_s = \int_0^\Omega L_{s\Omega} d\Omega = \int_0^\alpha L_{0s\Omega} e^{-kc(r-r_0)} \frac{da}{r^2}$$

$$= \int_0^\alpha L_{0s\Omega} e^{-kcd/\cos\alpha} \frac{2\pi r \sin\alpha \, r d\alpha}{r^2} \qquad (2)$$

where α is the angle of divergence of the light (one half the angle subtended at the slit by the monochromator lens). If α is small we may put

$$1/\cos\alpha = 1 + \delta \qquad (3)$$

and expand the exponential, discarding powers of δ higher than the second. We then get

$$L_s = 2\pi L_{0s\Omega} \delta e^{-kcd} \left(1 - \frac{kcd + 2}{2}\delta\right) \qquad (4)$$

The total light transmitted by the cell will be the sum of the transmitted light from all the elements, ds, of the slit.

$$L = \Sigma L_s ds = 2\pi \Sigma L_{0s\Omega} ds \delta e^{-kcd} \left(1 - \frac{kcd + 2}{2}\delta\right) \qquad (5)$$

If the light strikes no surface before reaching the rear cell wall

$$L = 2\pi\delta e^{-kcd} \left(1 - \frac{kcd + 2}{2}\delta\right) \Sigma L_{0s\Omega} ds \qquad (6)$$

$$ln \frac{L_0}{L} = ln \frac{2\pi\delta(1 - \delta)\Sigma L_{0s\Omega} ds}{2\pi\delta \left(1 - \frac{kcd + 2}{2}\delta\right) e^{-kcd} \Sigma L_{0s\Omega} ds}$$

$$= kcd \left(1 + \frac{\delta}{2} + \frac{kcd + 2}{8}\delta^2 + \cdots\right) \qquad (7)$$

The last step was obtained by expanding the logarithm. Passing to common logarithms, we obtain

$$\log\frac{L_0}{L} = \epsilon cd\left(1+\frac{\delta}{2}+\frac{2.303\epsilon cd+2}{8}\delta^2+\cdots\right) \qquad (8)$$

where ϵ is the extinction coefficient. The power series in δ readily gives the correction factor to apply in calculating ϵ. It may be seen that an angle $\alpha = 11°$ will make a correction of only 1 per cent. This angle roughly corresponds to a lens of aperture f 1.3, such a one as is not yet available for use in photochemical work. For a particular instance in practice α was 6°36′. δ was therefore 0.0067 and the error in measurement amounted to 0.34 per cent. It is thus seen that for the ordinary monochromator, δ is negligible and that we may use the formula

$$\log L_0/L = \epsilon cd \qquad (9)$$

Consideration of reflection correction has been omitted as it cancels out in the ratio of light quantities (the refractive indices of solvent and solution ordinarily do not differ much).

TABLE 1. *Galvanometer deflections*

D_2	D_1	Av D_2	Δ
110.5	44.6	110.6	66.0
110.7	45.0	110.9	65.9
111.0	45.3	111.0	65.7
111.0	45.6	111.5	65.9
112.0	45.9	111.8	65.9
111.7			65.88 Av.

D_2 = max. of galv. swing on opening shutter
D_1 = min. of galv. swing on closing shutter
Av D_2 = Average of two consecutive values of D_2. This is the value of D_2 at time of D_1 reading.
Δ = Av $D_2 - D_1$

To use this method it is necessary to measure instead of intensities the total amount of light transmitted by solution and solvent. This is readily done by integrating the light intensities measured by a line thermopile which, always at the same distance behind the rear cell wall, is moved at regular space intervals (2 mm) *across* the path of the emergent beam. The cross section of the latter must be small enough to be completely covered by the pile by its motion from one

side to the other. A sample set of measurements on 0.1 *M* KNO_3 follows. Table 1 gives the thermopile data for one of the galvanometer deflections recorded in Table 2.

TABLE 2. *Intensity integration*

H_2O	λ270mμ		0.1 *M* KNO_3
Δ	Δ/Δ_f	Δ	Δ/Δ_f
68.12_f			
2.06_{38}	0.0303	65.88_f	
5.30_{40}	0.0780	1.80_{40}	0.0273
7.76_{42}	0.1146	2.74_{42}	0.0416
6.12_{44}	0.0905	2.02_{44}	0.0306
5.72_{46}	0.0846	1.78_{46}	0.0270
5.94_{48}	0.0880	1.76_{48}	0.0267
0.62_{50}	0.0092	66.06_f	$0.1532=K$
67.24_f	$0.4952=K_0$		

$K=\Sigma\Delta/\Delta_f$

Δ_f = average deflection of thermopile when at a position immediately behind the monochromator slit.

Numerical subscripts denote position of pile in mm in its course across the path of the emergent beam.

The sum K is proportional to the total light transmitted and may be substituted in Eq. (9).

$$\log 0.4952/0.1532 = 0.1 \times 3.15 \times \epsilon$$

$$\epsilon = 1.62 \text{ (for } 270\text{m}\mu\text{)}$$

This checks with an interpolation for λ270mμ of data on KNO_3 (1.623) obtained by Halban and Ebert.[2]

I wish here to thank Prof. E. C. Kemble of Jefferson Physical Laboratory for several helpful suggestions.

SUMMARY

The divergence of the light beam from a monochromator slit introduces less than 1/2 per cent error in extinction coefficient measurements if the ratio of the total light transmitted rather than the ratio of the intensities is used in the calculations.

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CAMBRIDGE, MASSACHUSETTS.

[2]Halban and Ebert, Z. Physik. Chem., *112*, pp. 321–58; 1924.

THE EFFECT OF X-RAYS UPON THERMOLUMINESCENCE

By Frances G. Wick

During the course of an investigation[1] of the cathodoluminescence of fluorite in which some preliminary experiments were made to test the effect of radiations of various kinds, exposure to x-rays was found to produce a very marked change in the thermoluminescence of fluorite and other materials. Some specimens having no natural thermoluminescence acquired this property as a result of the exposure, others did not. Of the naturally thermoluminescent specimens examined, a few were changed little or not at all by the exposure but many were greatly changed in intensity and color of luminescence as well as in rate of decay. Crystals of fluorite and calcite showed such marked effects that a detailed study of a number of specimens was made.

Specimens of fluorite which were not naturally thermoluminescent became so to a greater or less degree upon exposure to x-rays. Crystals which were naturally active had the intensity at a given temperature greatly increased and many of them acquired, in addition to the luminescence of the original color, a new and entirely differently colored luminescence which flashed up before the thermoluminescence characteristic of the natural crystal appeared. Many of the most brilliant specimens had such rapid variations in color as to make quantitative measurements impossible. A number of specimens were found, however, in which there were no rapid color changes with a comparatively slow rate of decay and with sufficient intensity to make photometric observations possible.

A study of the following is included in this paper:

The effect of variation in temperature upon rate of decay of

(a) natural specimens.

(b) natural specimens after exposure to x-rays.

(c) specimens heated to remove all thermoluminescence then exposed to x-ray.

The effect of time of exposure to x-rays upon

(a) maximum brightness of thermoluminescence.

(b) rate of decay.

The effect of allowing specimens to stand for varying lengths of time after exposure to x-rays before observations were made.

[1]Frances G. Wick, Phys. Rev., *2*, *24*, p. 272. Wick and Gleason, J. O. S. A. & R. S. I., *6*, p. 369; 1924.

Since it is known that the size of the grain has an effect upon the brightness and the rate of decay of thermoluminescence, specimens were reduced to a standard condition by powdering them in a mortar and sifting them through a wire gauze of mesh seventy to the inch. This material was heated upon a copper plate with a platinum-copper thermo-junction inserted in it from which the temperature was read upon a Leeds and Northrup Thermocouple Pyrometer. The copper plate was heated upon a circular electric stove 2½ inches in diameter to which it was firmly fastened. After the plate had reached some desired constant temperature the material to be observed, mounted on a thin copper holder in a layer of .5 mm thickness, was laid upon it. Photometric observations of the brightness of thermoluminescence were made by the method commonly employed in the study of the decay of phosphorescence since thermoluminescence is of the same nature as phosphorescence appearing within certain temperature limits.

A diagram of the arrangement of apparatus is shown in Fig. 1.

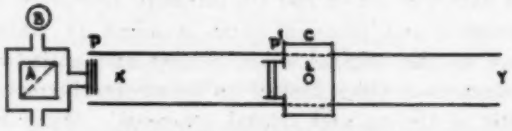

FIG. 1. *Arrangement of apparatus.*

A Lummer-Brodhun cube, A, was placed on a track, XY, about 3½ meters long. The stove upon which the specimen was heated was placed at an angle of about 45° opposite the face of the cube at B. A comparison lamp, L, in a movable carriage, C, ran on the track, XY, on which, at intervals of 25 cm, stops were placed. Absorption plates were inserted at P and P' to obtain a comparison source of the proper color and intensity. A chronograph was used to record time. The specimen was placed on the stove and the comparison lamp was moved so as to match the specimen at its maximum brightness. The time was recorded on the chronograph and this was taken as the zero of time. The carriage was then moved back to the next stop and allowed to remain until a match was again obtained. The time was recorded and the carriage moved back to the next stop. This procedure was continued until the end of the track was reached or until the specimen was too dim for observation. Curves were plotted in the usual way with the reciprocal square root of intensity as a function of time.

Exposure to x-rays was made as follows: a tungsten target Coolidge tube of the "bedside" variety was mounted so that the rays were directed downward through an opening about 5 cm in diameter in a lead glass shield. The material to be exposed to x-rays was placed in a thin layer directly under this opening at a distance of 11 cm. The tube was run on 2 milliamperes and 45 kilovolts.

A series of observations was made upon fluorite from Franklin County, New Jersey, which, in the natural condition, had a strong green thermoluminescence the duration of which was sufficiently long to make observations of decay possible. After exposure to x-rays, upon heating the specimen, a yellow luminescence appeared, which came up before the green did and lasted a very short time, disappearing before the green became visible. Specimens heated until all thermo-

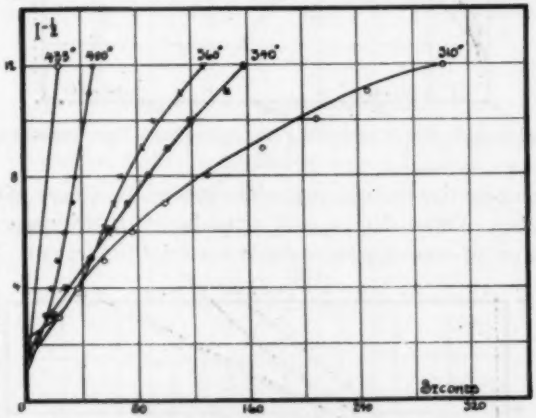

FIG. 2. *Natural Franklin fluorite. Decay curves at different temperatures.*

luminescence had disappeared showed, when heated, both the yellow flash of short duration and the more persistent green. The yellow luminescence is not permanent but the green was still bright after an interval of ten months.

It seems entirely possible that the yellow color is due to a different impurity in the crystal from that which causes the green and that this becomes an activator only after exposure to x-rays. It is known from the cathode-ray spectra of the fluorites[2] that many single specimens contain as impurities a number of rare earths.

Decay curves for the natural material at different temperatures are shown in Fig. 2, the green maximum being taken as the zero of

[2]Frances G. Wick, Phys. Rev., *2*, *24*, p. 272; 1924.

time. Fig. 3 shows a corresponding set of curves for material which was heated until all thermoluminescence had disappeared, then exposed to x-rays. A set of decay curves at different temperatures was

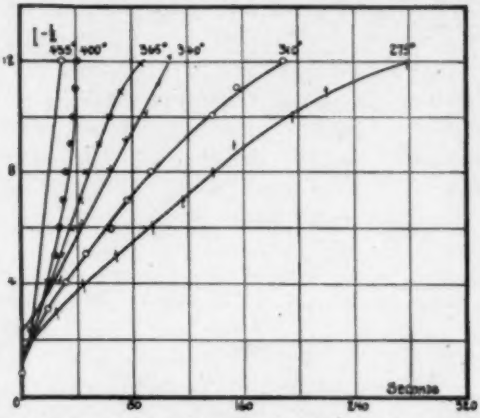

FIG. 3. *Heated Franklin fluorite exposed to x-rays for one hour. Decay curves at different temperatures.*

also made upon the natural material exposed to x-rays without previous heating. These curves (not reproduced in this paper) showed that the effect of exposing the natural material to x-rays is, in general,

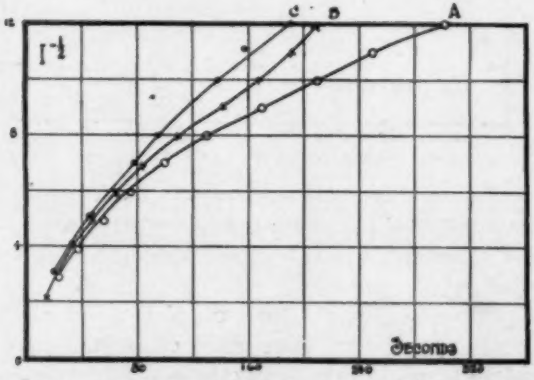

FIG. 4. *Franklin fluorite. Decay curves at 310°C. A—natural; B—natural material exposed to x-rays one hour; C—heated material exposed to x-rays one hour.*

to increase the brightness at any given temperature and make the decay more rapid. Decay curves of Franklin fluorite differently treated and observed at a temperature of 310°C are shown in Fig. 4.

Curve *A*, for the natural, unexposed material shows a slower rate of decay than that shown in Curve *B* for the natural material after exposure of one hour to x-rays. The decay of the material heated to

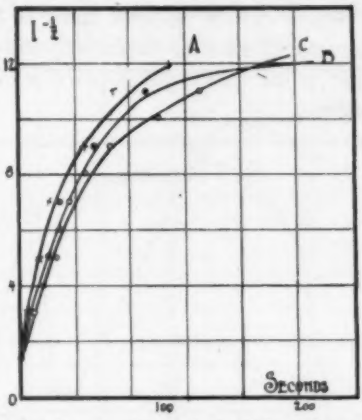

FIG. 5. *Natural Franklin fluorite exposed to x-rays for different lengths of time. Decay curves at 105°C. A—exposure 1 hour: B—exposure 10 hours: C—exposure 20 hours.*

exhaust all thermoluminescence then exposed for one hour, Curve *C*, is the most rapid.

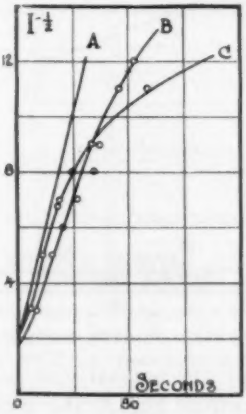

FIG. 6 *Heated Franklin fluorite exposed to x-rays for different lengths of time. Decay curves at 105°C. A—exposure 1 hour: B—exposure 10 hours: C—exposure 20 hours.*

It will be observed from a comparison of Figs. 2 and 3 that the treated specimen, at any given temperature, has a more rapid decay than the natural material. It is of interest to observe that the decay

curves at the lower temperatures (Fig. 2 and Fig. 3) are of the type described by Nichols and Howes[3] as "persistent." As the temperature of observation rises the curves gradually change over the "vanishing" type. These sets of curves resemble those of Ives and Luckiesh[4] for the phosphorescence of alkaline earth sulphides in which a similar change in the shape of the curves takes place with variation of temperature of observation.

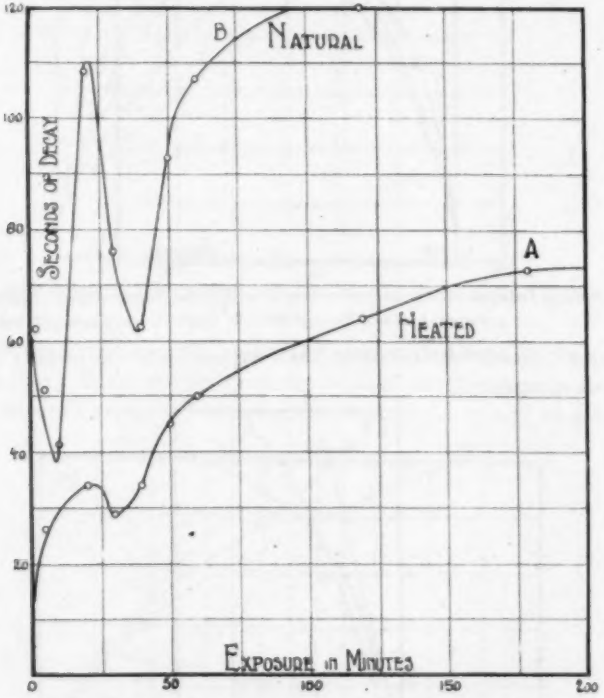

FIG. 7. *Franklin fluorite. Length of time required to decay to a definite brightness as a function of time of exposure to x-rays. A—heated material: B—natural material.*

The effect of exposing the natural material to x-rays for different periods of time is shown in Fig. 5, the temperature of observation being 105°C. Curve *A*, taken after an exposure of 1 hour does not differ greatly from curves *B*, and *C*, taken after exposures of 10 and 20 hours respectively. Fig. 6 shows a similar set of curves taken at

[3]Nichols and Howes, Nat. Acad. Sci., Proc., *4*, pp. 305–312.
[4]Ives and Luckiesh, Astrophys. Jour., *36*, p. 330; 1912.

the same temperature upon material which has heated to remove all natural thermoluminescence then exposed for different periods of time to x-rays. It will be observed that these curves resemble those of Fig. 5 in their relation to each other but the decay is much more rapid than in the case of the unheated material.

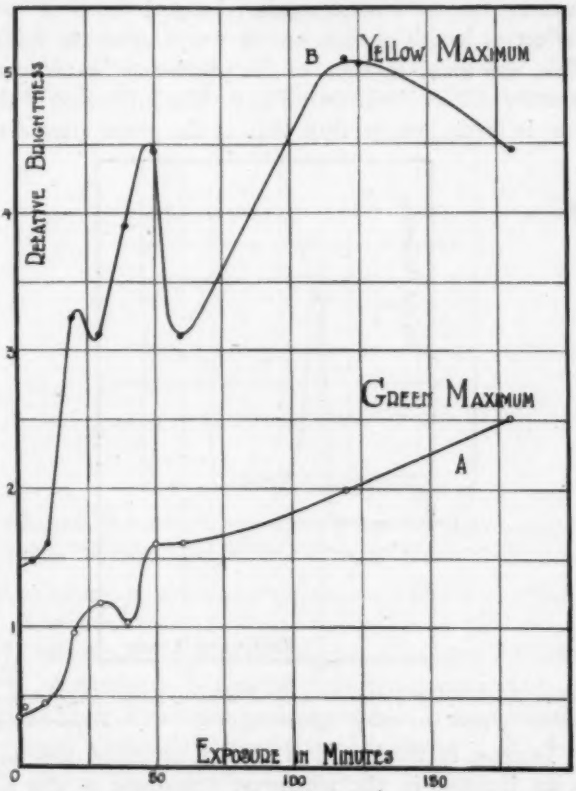

FIG. 8. *Heated Franklin fluorite. Brightness of maxima as function of time of exposure to x-rays. Temperature of observation 205°C. A—green maximum: B—yellow maximum.*

Measurements were made upon the length of time required for natural and heated specimens to decay to a definite brightness—that which matched the intensity of the comparison lamp when it was placed at the last stop on the track—as a function of time of exposure to x-rays. The temperature of observation was 205°C. It will be observed from Fig. 7 that, in all cases, the previously heated

material takes a shorter time than the natural material and that both curves indicate great irregularity. An increase in time of exposure to x-rays does not always lengthen the time required to decay to the given intensity. There are maxima and minima in both curves for intervals of exposure under one hour. Exposures of more than one hour seem to produce a more regular change in both curves.

The effect of length of exposure to x-rays upon the brightness of the yellow and green maxima of the previously heated material at a temperature of 205°C is shown in Fig. 8. The brightness of the yellow maximum is much greater than that of the green after a given ex-

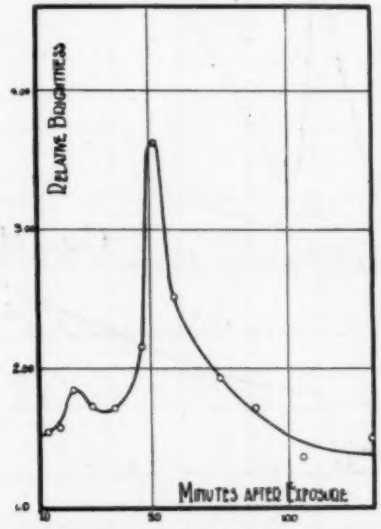

FIG. 9. *Natural Franklin fluorite exposed to x-rays 30 minutes. Temperature of observation 260°C. Brightness of green maximum at different lengths of time after exposure to x-rays.*

posure. Increase in the length of time of exposure does not alway. produce an increase in the maximum brightness of the specimens The green maximum is more intense after an exposure of 30 minutes than it is after an exposure of 40 minutes. Curve *A*, Fig. 7 shows a point of inflection at about the same period of exposure as the point of inflection shown in Curve *A*, Fig. 8.

Observations were made upon the relative brightness of the green maximum at different periods of time after exposure of 30 minutes to x-rays. The temperature of observation was 260°C. Fig. 9 shows that the brightness of the maximum increases during a period of

fifty minutes in which the specimen was allowed to stand in the dark after exposure, then the brightness decreased. This same effect was observed in a number of other samples of fluorite. Measurements were made upon chlorophane, a thermo-luminescent fluorite obtained from the late Dr. Charles Baskerville. This material had a natural blue-green thermoluminescence. Exposure to x-rays introduced a white flash which came up before the green color when the specimen was heated. Fig. 10 shows the effect upon the brightness of this white flash of allowing a specimen, previously heated to remove natural

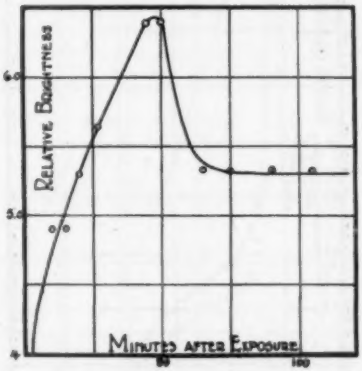

FIG. 10. *Chlorophane exposed to x-rays 75 minutes. Temperature of observation 420°C. Brightness of white maximum at different lengths of time after exposure to x-rays.*

thermoluminescence, to stand after exposure to x-rays. The temperature of observation was 420°C and the length of time of exposure to x-rays 75 minutes. A distinct maximum is reached 50 minutes after exposure. A specimen of fluorite from Westmoreland, N. H. which had a green flash introduced by exposure to x-rays was found to have very irregular variations in maximum brightness of this flash within the first hour after exposure to x-rays as is shown in Fig. 11. This specimen was one of the most interesting of the fluorites but the changes in color of thermoluminescence upon heating made the measurement of decay impossible. The natural thermoluminescence was brilliant purple consisting of four distinct bands which could be observed through a small direct vision spectroscope. The approximate limits of these bands were 5750–6250A; 5200–5350A; 4750–4950A; faint band barely visible of shorter wave length.

The green thermoluminescence which resulted from bombardment of this same fluorite with x-rays consisted of two distinct line-like maxima at wave lengths 5568 and 5390A. This same green color is observed under cathode ray excitation and it is due to terbium. This specimen evidently has a number of different impurities which may account for the very irregular variation in intensity of the green maximum after exposure to x-rays. Material exposed to x-rays was examined eight months later and found to still retain the green luminescence imposed by x-rays.

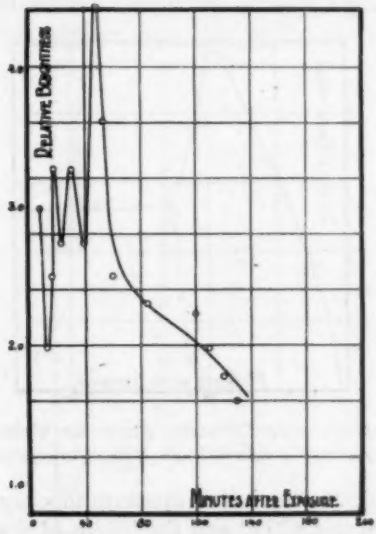

FIG. 11. *Fluorite from Westmoreland, N. H., exposed to x-rays 1 hour. Temperature of observation 263°C.*

Observations were made of three different varieties of calcite kindly furnished by W. P. Headden, of the Colorado Agricultural College, Fort Collins, Colorado. Calcite from an old Crookes tube was also examined. Exposure to x-rays increased the intensity of the thermoluminescence of all the calcites observed—the color of this luminescence was orange. The calcite from Ingleside was given, by exposure to x-rays, a green luminescence preceding the orange. Calcite from Fort Collins was most interesting. It had a natural orange thermoluminescence. After exposure to x-rays an additional luminescence was observed. When the specimen was heated two distinct flashes came up. The first flash was observed to be a band with limits about 5780 to 6800A. After

this had died down a second thermoluminescence appeared the limits of which were about 5250 to 6500A.

Measurements were made upon the brightness of the maxima of the first and second flashes immediately after exposure as a function

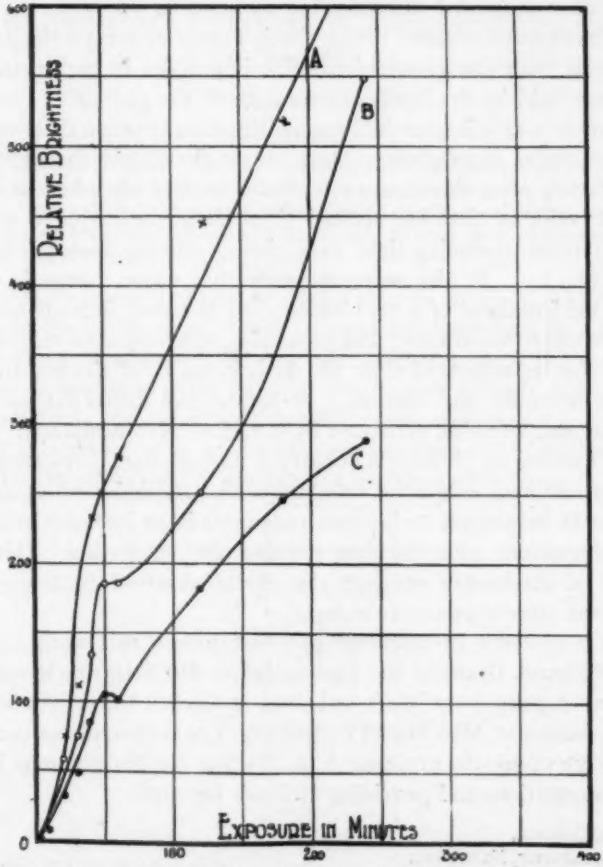

FIG. 12. *Fort Collins calcite. Brightness of maxima as function of time of exposure to x-rays. Temperature of observation 275°C. A—first flash immediately after exposure: B—second flash immediately after exposure: C—second flash 50 hours after exposure.*

of the time of exposure to x-rays, curves *B* and *C*, Fig. 12. Curve *A* shows the second flash 50 hours after exposure. The first flash had disappeared by this time. These curves show breaks after an exposure of about 50 minutes.

The irregularities occurring in the curves for fluorite and calcite suggest a mechanism which is very complicated. In order to explain the fact that intensity of maximum luminescence is greater after the specimens have been allowed to stand than immediately after exposure to x-rays, Dr. Wilber suggested the following as being in line with the theory of Leonard and Saeland:[5] Under the influence of x-rays electrons are separated from the atoms of metallic impurities in the crystals and these are held in the insulating medium of the phosphor. After the exposure to x-rays is over the force of attraction between these electrons and the metal atoms brings them nearer the atoms than they were immediately after the exposure to x-rays so that when heat is applied after a period of time has elapsed since the exposure there are more combinations producing light than there could be immediately after exposure. . . . . It also seems possible that some change is started under the influence of x-rays which does not stop immediately even after exposure has ceased. The possibility of a very slow radioactivity might also be suggested since the flash in many of the materials examined resembles that described by Karrar and Kabakjian[6] as taking place in zinc sulphide activated by a radio-active compound.

The flashing up phenomena observed during phosphorescence decay by Ives and Luckiesh[7] after exposure of zinc sulphide and other phosphorescent substances to infrared radiations is an instance of another effect dependent upon the time elapsing since excitation. This flashing up of luminosity suggests the effects observed in some of the specimens after exposure to x-rays.

It is a pleasure to acknowledge assistance in this work from the Ellen Richards Grant of the Association to Aid Scientific Research by Women. A part of the Work was done at Cornell University with the able assistance of Miss Mabel K. Slattery. The author wishes to express her sincere thanks to Professor E. L. Nichols for his generous help in giving suggestions and providing facilities for work.

VASSAR COLLEGE,
POUGHKEEPSIE, NEW YORK.

[5]Lenard and Saeland, Ann. Physik., *28*, pp. 476–502, 1909.
[6]Karrer and Kabajian, Jour. Frank. Inst., *186*, p. 317; 1918.
[7]Ives and Luckiesh, Astrophys. Jour., *34*, p. 173; 1911.

INSTRUMENT SECTION

GRÜNEISEN'S CRITERIA FOR THE CAPILLARY VISCOSIMETER

By N. Ernest Dorsey

ABSTRACT

It is shown that Grüneisen's criteria, for determining the velocity at which the value of pt for a given viscosimeter will exceed, by a given amount, its value at very low velocity, are incorrect. Other criteria, in accordance with the general experience of investigators and with Bond's observations at very low velocities are derived. An error which vitiates many of the published tests of viscosimeters is pointed out.

In 1904, Grüneisen[1] published a set of criteria for use in the designing of capillary viscosimeters. These criteria have been used by a number of investigators; and are still being used. Hence, even at this late date, it seems desirable to point out that the manner of their derivation is unsatisfactory, and the criteria themselves are erroneous.

In the following, p is used to denote the over-all driving pressure, t the time required for the passage of a certain fixed volume (V) of the

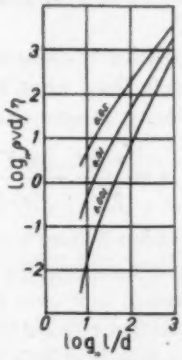

Fig. 1. *Grüneisen's criteria.*

liquid, l the length, l_e the effective length, and $d(=2r)$ the diameter of the capillary, $v(=V/\pi r^2 t)$ the velocity of efflux, $\delta=[pt-(pt)_0]/(pt)_0$ where $(pt)_0$ denotes the value of pt when v is infinitely small, v_δ = value of v which corresponds to δ, η = viscosity, ρ = density, and m is a numerical coefficient.

[1] Grüneisen, E., Wiss. Abh. Physik. Techn. Reichsanstalt, *4*, pp. 151–184; 1904.

The criteria refer to viscosimeters in which a straight, cylindrical capillary of circular cross-section connects two relatively large reservoirs of liquid. They consist of three curves, each purporting to be the locus of the points ($x = \log l/d$, $y = \log dv$) which correspond, for water at 10°C, to a particular value of δ. From such loci it is possible to determine for any value of dv the smallest value l/d can have if pt is to differ from $(pt)_0$ by not more than $\delta(pt)_0$. The values of δ represented are 0.001, 0.01, and 0.05. An equation representing the curve for $\delta = 0.001$ is given. By suitably shifting the origin of ordinates by an amount corresponding to the value of $\log_{10}\rho/\eta$ for water at 10°C, and changing in a corresponding manner the designation of the ordinates, the curves become freed from all dependence upon a particular substance. Thus shifted, they are shown in Fig. 1; on each curve is marked the corresponding value of δ.

The criteria rest solely upon the assumption that throughout the range of velocities considered the variation of pt with t is represented by Eq. (1),

$$pt = ae^{b/t} \tag{1}$$

and upon the values of a, b, and n which were derived by Grüneisen, in an unexplained manner, for 10 of Poiseuille's tubes and for one of his own, all for water at 10°C.

The assumption itself is distinctly disturbing, since recorded observations quite generally indicate that, when δ exceeds a few per cent, pt varies linearly with the velocity, in accordance with Eq. (2),

$$\eta = \frac{\pi r^4 pt}{8Vl_e} - \frac{m\rho V}{8\pi l_e t} \tag{2}$$

while the exponential expression defines a very different type of variation. Hence the latter can not express the actual variation of pt, but at best can merely approximate to it over a limited range of t. Under such conditions, an extrapolation beyond the range within which data are available for testing the degree of fit is quite untrustworthy.

But it is exactly such extrapolation that is used in deriving the criteria. There is no observation at $v = 0$, the datum point from which δ is measured, and, of the tubes considered, there was but one for which there was any observation at a velocity so low that $\delta = 0.001$; for that tube there were four such observations, but the mean of these four does not lie upon the assigned exponential curve. Hence, all values of $(pt)_0$, and everything upon which the curve for $\delta = 0.001$ is based, were

obtained by extrapolation. Furthermore, the criterion for $\delta = 0.001$, which rests upon no experimental data whatever, is the one upon which the most stress is placed, and is the one which seems to have been the most used. A curve of an arbitrarily assumed type has been fitted to the observations at moderate velocities, and the uncontrolled lower portion has been used for extrapolating to zero velocity, for obtaining the datum point from which δ is measured. Every error in this datum point affects the value of δ computed for a given velocity. Furthermore, some of the values assigned to n are less, and others are greater, than

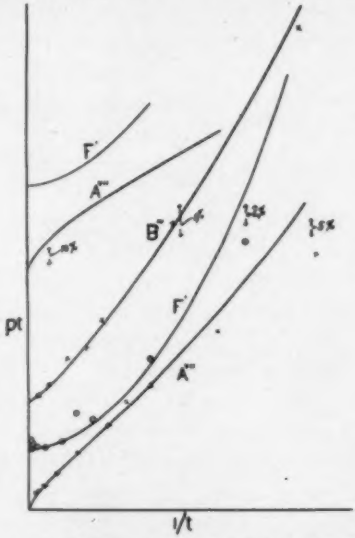

FIG. 2. *Grüneisen's exponential curves for three of Poiseuille's tubes. The origin of pt and the scale of pt and of 1/t vary from curve to curve. All of Poiseuille's recorded observations with these tubes are indicated. The lower portion of the A^{vii} curve is repeated on a larger scale in the upper portion of the figure, and is accompanied by a repetition of the lower portion of the F^{i} curve.*

unity; this gives rise to two types of curves. Near the origin, some of the curves are concave downwards and others are concave upwards. Surely such differences are not to be expected, and should not be accepted without very good reason. None is given. Three typical curves computed from the parameters given by Grüneisen are shown in Fig. 2, together with the observed points. For A^{vii}, n is less than unity; the curve begins with concavity downwards, then straightens out, and finally becomes concave upwards. In such cases, it is the approximately straight portion which has been fitted to the observa-

tions. It will be noticed that the upper portion of this curve is much too steep; the observation at the highest velocity lies over 15 per cent below the curve. The low position of the intercept of this curve is not justified by the observations; it imposes upon the criterion a very low velocity for any assigned value of δ. There seems to be no justification for fitting to the A^{vii} observations anything except a straight line.

For the other two curves, n is greater than unity. Again it is difficult to understand why it was considered that an exponential curve fits the B^{iv} observations significantly better than a straight line does. If the observations with the F^{i} tube stood alone, a curved line might be considered justifiable; but the observations with this tube are plainly erratic, and the amount of curvature which they suggest is excessive as compared with any that can fairly be claimed to be indicated by the observations with the other tubes. Under such conditions, this tube should be ignored in any study connected with the derivation of general relations; but after such relations have been established, the observations with it should be examined for the purpose of ascertaining whether they accord with the established relation as closely as their errors justify one in expecting. This is the only tube studied by Grüneisen for which observations at velocities as low as that corresponding to $\delta=0.001$ were available; these observations are represented by the extreme four points, and it will be noticed that the curve passes distinctly below their mean position. It will also be noticed that by the time the observation at the highest velocity is reached the curve is over 4 per cent too high, and is rapidly becoming steeper. Actually, the curve fits the observations only very little better than a straight line would. The curves for the other eight tubes are no more satisfactory than the three just considered.

Furthermore, Eq. (2) imposes upon the criteria certain conditions which are not met by the criteria given by Grüneisen. It is generally assumed that $(pt)_0=8V\eta l_e/\pi r^4$, but this can be true only if the Couette correction has the same value when the velocity is infinitely small as it has at moderate velocities. This is not necessarily the case; indeed it is not true,[2] but the percentile difference so produced in the effective length is small, and decreases as l/r increases. Hence if l/r is not too small, it may be neglected, and then it follows from Eq. (2) that $\delta=rmR/16l_e$, where R denotes $\rho vd/\eta$. Taking logarithms and differentiating under the condition that δ shall be constant, it is found that $d(\log l_e/r)/d(\log R)=1$; that is, when the data are plotted as in Fig. 1

the curves, for large values of l/r, approximate to straight lines with a slope of 45°. This is not true of any portion of the curves in that figure. This discrepancy was noted by Grüneisen, but appears to have been interpreted as evidence that the 45° slope does not accord with the observations. Such interpretation is unwarranted, as it involves the discarding of Eq. (2).

Without restricting l/r to being great, we may write $8V\eta l_o/\pi r^4 = (pt)_0(1-\lambda r/l)$, where λr is a small length. Then

$$\delta = -\frac{\lambda r}{l} + \frac{V\eta mR}{2\pi r^3(pt)_0}$$

and for any given, fixed value of l and r

$$\log_{10}\frac{\delta_1}{\delta_2} + \log_{10}\left\{1 - \frac{\lambda r(\delta_1 - \delta_2)}{l\delta_1\delta_2}\right\} = \log_{10}R_1 - \log_{10}R_2 \qquad (3)$$

As the quantity to the right of the equality sign is the vertical distance between the curves for δ_1 and δ_2 in Fig. 1, Eq. (3) shows that this distance should equal $\log_{10}\delta_1/\delta_2$, if l/r is great; and if l/r is small and $\lambda>0$, this distance should be less than $\log_{10}\delta_1/\delta_2$, and should decrease as l/r decreases. For positive λ, the graphs should crowd together as l/r decreases. It will be noticed that for large values of l/r the curves in Fig. 1 are much closer together than $\log_{10}(\delta_1/\delta_2)$, and the distance between them increases as l/r decreases. It is not evident how the first can be reconciled with Eq. (2); the second demands that λ be negative, that the Couette correction at infinitely small velocities shall be less than at moderate velocities. From Bond's recent observations it is found[2] that the reverse is true; hence λ must be positive.

Thus it is evident that the manner in which the criteria were derived is far from satisfactory, is not such as to commend the criteria; and the latter are, at least in some respects, at variance with the general experience of investigators in this field.

Until Bond's[3] investigations in 1921 and 1922, there were no satisfactory experimental data for the rate of variation of pt with the velocity when the latter is very small. His observations showed that if the capillary is thick walled, has square-cut ends with smooth, sharp edges, has a lumen which is a circular cylinder, and connects two large reservoirs of liquid, then, if the velocity does not exceed a certain critical

[2] Dorsey, N. E., Phys. Rev., (2), *28*, p. 833-845; 1926.

[3] Bond, W. N., Proc. Phys. Soc. London, *33*, p. 225; 1921; *34*, pp. 139–144; 1922.

velocity (v_c) defined by a certain Reynolds number, pt is constant, is independent of v; and at higher velocities, pt varies linearly[2] with v as indicated in Fig. 3. It has been shown[2] that such a variation might have been predicted from quite elementary considerations, and that under usual conditions the value of pt is given by Eqs. (4) and (5)

$$v \leqq v_c \qquad pt = (pt)_0 = \frac{8\eta V}{\pi r^4}(l + \epsilon_1 + \epsilon_2) \tag{4}$$

$$v > v_c \qquad pt = \frac{8\eta V}{\pi r^4}(l + \epsilon_1) + \frac{m\rho V v}{\pi r^2} \tag{5}$$

where ϵ_2 is the value of the term containing m when $v = v_c$. For the ideal case, $m = 1.00$ and $\epsilon_1 = \epsilon_2 = \rho r^2 v_c/8\eta$, and when $v < v_c$, the Couette correction is twice as great as it is when $v > v_c$. For the ideal case, Eq. (6)

$$\frac{m\rho vd}{\eta} = 32\delta\left[\frac{l}{d} + \frac{\epsilon}{r}\left(1 + \frac{1}{2\delta}\right)\right] \tag{6}$$

may readily be derived from Eqs. (4) and (5), and ϵ may be replaced by its value ($0.573r$) derived from Bond's observations.

This equation is the criterion which Grüneisen sought, but it is significantly different from the equation which he gave for $\delta = 0.001$,

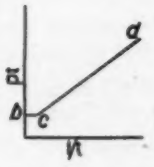

FIG. 3. *Actual type of the variation of pt with 1/t.*

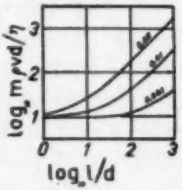

FIG. 4. *Criteria for the ideal case.*

and its graphical representation (Fig. 4) is quite different from the curves he derived (Fig. 1). If the two sets of curves were suitably superposed, the curves for $\delta = 0.001$ would intersect near $l/d = 140$; those for $\delta = 0.01$, near $l/d = 90$; and those for $\delta = 0.05$, near $l/d = 65$. For any smaller value of l/d the corresponding Grüneisen criterion calls for an excessively low velocity; for any larger value, it permits a velocity which is too great—it may be several times too great. The

former error will do no serious damage, as it merely discourages the use of short tubes; but the latter, and here I am considering especially the curve for $\delta = 0.001$, may lull one into a false sense of security when long tubes are used.

Equations (4), (5) and (6) quite generally apply when the instrument fulfills the conditions realized by Bond. But if the tube is very long, if the instrument is subjected to only very slight mechanical disturbances, and if the liquid is being rapidly accelerated as the critical velocity (v_c) is being passed, then Eq. (4) may continue to be satisfied[2] at velocities much greater than v_c. Under this exceptional combination of circumstances, δ may be zero even though the velocity is much higher than that defined by the criterion for $\delta = 0.001$.

It may be asked: If the Grüneisen criteria are actually as much in error as is here indicated, how is it that a number of observers who have actually tested the behavior of viscosimeters designed in accordance with them have found the instruments to be satisfactory? Those instruments for which l/d was less than the value at which the criterion is correct should have been satisfactory, as they were being used at an unnecessarily small velocity. Also, the design was generally such that the velocity was less than that specified by the criterion; this would tend to compensate for the error when l/d exceeded the value at which the criterion is correct. But more especially is the explanation to be sought in errors in the performance of the test. In many cases the pressure was incorrectly computed. The test consists in observing the time of efflux of a fixed volume of liquid for each of a series of different driving pressures, the variation in pressure being secured by applying to one side of the instrument a series of constant external pressures, positive or negative. If the resulting variation in pt lies well below the value set by the criterion, the variation in p being great, it is concluded that the instrument is satisfactory. In each case it is the effective pressure—the constant pressure under which the efflux would require the same time—which should be used in computing pt. But, very frequently, observers have simply added to the applied external pressure a fixed correction; this is equivalent to adding a pressure corresponding to some fixed instrumental head. It may readily be shown that this procedure is incorrect. If the applied pressure is very great as compared with the change in the instrumental head during efflux, then the effective pressure is very nearly equal to the pressure at mid-volume—to the sum of the applied pressure and the

pressure which corresponds to the instrumental head when half of the fixed volume has passed through the capillary. If, on the contrary, the applied pressure is such that near the end of the efflux the resultant pressure is small as compared with the change in the instrumental head, then the time required for the efflux of the second half of the volume will be distinctly greater than that for the first, and consequently, the effective pressure will be less than the pressure at mid-volume. As the applied pressure decreases, the instrumental head which must be added to it in order to obtain the effective pressure likewise decreases. Hence, if the pressure used in the computation of pt is obtained by adding to the applied pressure the equivalent of a fixed instrumental head, then the computed values of pt for the smaller values of p—that is, for the smaller velocities—will be too great as compared with those for the higher values. An error of this sign tends to compensate for the normal variation of pt with the velocity.

In one case in which this error was made and in which information incidentally contained in the published paper was such that the amount of the error could be computed with fair accuracy, the published values of pt were exceedingly constant; the average deviation from the mean of all was distinctly less than 0.05 per cent, and was just about equal to the amount by which the value of pt at the lowest pressure exceeded that at the highest. But it was found that the error in computing the pressure was such that had the true effective pressure been used, the value of pt at the lowest pressure would have been over 0.5 per cent lower than that at the highest, and the variation of pt with the velocity would have been more rapid than is usual, although we are told that the terminals of the tube had been made trumpet shaped, for the purpose of avoiding such variation.* For this tube, $\log_{10} l/d = 2.6$ and during the test $\log_{10} \rho v d/\eta$ was varied from 1.5 to 1.8.

This error in computing the pressure is found so frequently in those cases in which the published details are sufficient to reveal exactly what was done, that it is scarcely justifiable to assume that it is absent from work which is published in insufficient detail.

* Though a number of investigators have advised the use of trumpet shaped terminals as a means for securing constancy of pt, there seems to be no satisfactory experimental evidence that it is practically efficacious. Indeed, the fact that the jet entering the reservoir may continue for quite a distance with a diameter which is not appreciably different from that of the capillary from which it proceeds suggests that the angle of the trumpet must be exceedingly small if it is to produce the effect desired.

In conclusion, a word of caution seems desirable. The quantities ϵ and m in Eq. (6) depend upon the configurations of the terminals. As the quantity ϵ is small, its variation from Bond's value will probably be of little importance if the ideal conditions are closely approached, but even under these conditions m may vary widely, perhaps from 1 to 2. Hence caution should be exercised in the use of that equation.

NATIONAL RESEARCH COUNCIL,
WASHINGTON, D. C.

Données Numériques de Spectroscopie. By. L. Bruninghaus, D. Sc. (Taken from Vol. V of the Annual Tables of Constants and Numerical Data, 1917–1922) ix+352 pages. Cambridge University Press, London; University of Chicago Press, Chicago.

The general scope of the Annual Tables of Constants and Numerical Data is too well known to need comment. Recognizing the growing need of having available for spectroscopists, complete numerical data in concise and convenient form, the International Committee in charge of the Tables has reprinted separately the spectroscopic data of volume V, which covers the years 1917–1922, inclusive.

Chapter I, containing over two hundred pages of data on emission spectra, is divided into five sections, viz: Primary and secondary standards of wave length; arc flame and spark spectra; x-rays; luminescent spectra; stellar spectra. Chapter II(13 pages) gives data on the Zeeman, Stark and Einstein effects. Chapters III, IV and V (35 pages) give recent data on the fine structure of spectral lines, and the effects on spectra of pressure, temperature, and other conditions of excitation. The grouping into series, of lines and of emission or absorption bands occupies chapter VI (23 pages). The remaining four chapters are devoted to the absorption spectra of inorganic gases and vapors; organic gases and vapors; inorganic solids, liquids and solutions; and organic solids, liquids and solutions, respectively. Much of the data on absorption is conveniently given in the form of graphs.

The book is well indexed, both by authors and substances. Any comment on the value of such a compilation is superfluous, covering, as it does, the work of over three hundred authors in some forty-seven journals. It speaks for itself. And it will be appreciatively received by those whose work lies in the field of spectroscopy.

F. K. R.

Multiple small-angle scattering of hydrogen nuclei. Homma attempted to perform with H nuclei (canal-rays) experiments upon multiple small-angle scattering similar to those performed in great numbers some twenty years ago upon He nuclei (alpha-particles). A beam of canal-rays after traversing a thin gold foil becomes divergent, as was shown by allowing it to fall upon a phosphorescent screen; the brightness of the diffuse fluorescent spot which it formed was measured at various distances from its center with a Lummer-Brodhun photometer, was assumed proportional to the number of particles striking per unit area, and the probability of resultant deflections through various angles was deduced as a function of angle from these measurements. It seems remarkable that photometric measurements with such a source are feasible; but the curves are mostly smooth. They show very clearly that there is a most probable angle of deflection, and this varies quite closely as the inverse third power of the speed of the particles (range: 30,000 to 50,000 equivalent volts) and directly as the 3/2 power of the thickness of the foil (range: .07 to .21μ). These laws of variation do not aggree with those deduced by Bothe (ZS. f. Phys. *4*, pp. 300–314; 1921) for small angle scattering of alpha-particles by massive atoms, but the problem is a difficult one to treat theoretically.—[E. Homma (German University, Prague); Ann. d. Phys., *80*, pp. 609–620; 1926].

KARL K. DARROW.

Distribution-in-angle of electrons scattered by helium. A primary beam of electrons was projected into helium gas from a hot filament through slits, and a secondary beam isolated by another pair of slits beyond which it entered an evacuated tube pervaded by a magnetic field for determining the velocity-distribution; apparently, however, this device served only for segregating first the electrons which had lost very little energy, and then the electrons which had lost 20 equivalent volts of energy. The amounts of electrons of these two classes, plotted versus angle of scattering from 2.5° to 90°, are reproduced in the note; they show curious lobes resembling those of Davisson's curves of scattering by metals. The curve for electrons which have suffered inelastic impacts show two maxima, between which a third appears when the energy of the primary electrons is increased past 200 equivalent volts; the angle at which this latter maximum occurs does not vary as the primary energy is further increased to 400, although the others shift (in opposite directions, to judge from the graphs). The electrons which have suffered no loss of energy produce a curve with only two maxima even at the highest value (400) of primary energy.—[E. G. Dymond (Princeton); Nature, *118*, pp. 336–337; 1926.]

KARL K. DARROW.

A GNOMONIC RULER FOR LAUE PHOTOGRAPHS

By Maurice L. Huggins[1]

A very useful ruler for making gnomonic projections of crystals from their Laue photographs has been described by Wyckoff.[2] The numbers read off the ruler are the distances of the Laue spots from the central image. These distances are used in the calculation of the wave lengths producing each reflection by means of the well known equation $n\lambda = 2\ d \sin\theta$, where n is the order of reflection, λ is the wave length, d is the distance between reflecting planes (in the first order) and θ is the angle of reflection. θ may be calculated from:

$$\tan 2\theta = \frac{\text{(distance of Laue spot from central image)}}{\text{(distance from crystal to plate)}}$$

These calculations may be made once for all, and a table constructed showing values of $\sin\theta$ corresponding to the distances of the spot from the central image. But when hundreds of wave length calculations are necessary, the time involved in this transformation is considerable, to say nothing of the chances for error.

The writer of this paper therefore recommends that in making the ruler it be calibrated to read $\sin\theta$ directly, and presents on the following page the data needed for its preparation.[3]

Gates Chemical Laboratory,
California Institute of Technology,
July 1, 1926.

[1]National Research Fellow at the California Institute of Technology, 1923–1925.

[2]Wyckoff, Am. J. Sci., *50*, p. 317; 1920; "The Structure of Crystals," Chem. Cat. Co., New York, p. 126; 1924.

[3]Two excellent rulers have been made for the writer from these data by Mr. F. C. Henson in the Laboratory of the Mt. Wilson Observatory, Pasadena, California.

TABLE 1. *Data for Preparing Gnomic Ruler*

Distance from Crystal to Plate, 5 cm.

cm to left of zero	Reading (sin θ)	cm to right of zero	cm to left of zero	Reading (sin θ)	cm to right of zero
0.705	0.07		3.337	0.29	16.50
.756	.075		3.413	.295	16.19
.808	.08		3.490	.30	15.90
.859	.085		3.569	.305	15.61
.911	.09		3.648	.31	15.34
.963	.095		3.731	.315	15.06
1.014	.10	49.80	3.814	.32	14.80
1.067	.105	47.36	3.898	.325	14.55
1.121	.11	45.17	3.982	.33	14.30
1.174	.115	43.20	4.070	.335	14.06
1.227	.12	41.37	4.158	.34	13.83
1.280	.125	39.68	4.255	.345	13.605
1.333	.13	38.15	4.341	.35	13.385
1.388	.135	36.70	4.437	.355	13.169
1.443	.14	35.35	4.534	.36	12.958
1.498	.145	34.12	4.634	.365	12.755
1.553	.15	32.96	4.734	.37	12.554
1.608	.155	31.87	4.838	.375	12.360
1.663	.16	30.87	4.943	.38	12.171
1.720	.165	29.89	5.051	.385	11.989
1.777	.17	29.00	5.160	.39	11.808
1.835	.175	28.12	5.277	.395	11.623
1.893	.18	27.33	5.393	.40	11.453
1.951	.185	26.55	5.512	.405	11.287
2.010	.19	25.84	5.632	.41	11.125
2.070	.195	25.14	5.761	.415	10.963
2.130	.20	24.50	5.889	.42	10.804
2.191	.205	23.87	6.024	.425	10.649
2.252	.21	23.27	6.160	.43	10.498
2.314	.215	22.71	6.303	.435	10.350
2.376	.22	22.17	6.446	.44	10.206
2.440	.225	21.65	6.600	.445	10.063
2.504	.23	21.15	6.753	.45	9.923
2.568	.235	20.68	6.916	.455	9.786
2.633	.24	20.23	7.079	.46	9.653
2.699	.245	19.79	7.255	.465	9.521
2.766	.25	19.37	7.431	.47	9.390
2.834	.255	18.96	7.620	.475	9.263
2.902	.26	18.57	7.808	.48	9.139
2.973	.265	18.19	8.013	.485	9.016
3.044	.27	17.83	8.218	.49	8.894
3.115	.275	17.48	8.439	.495	8.776
3.187	.28	17.15	8.660	.50	8.660
3.262	.285	16.82			

NATURAL LOGARITHMS OF LARGE NUMBERS*

By Irwin Roman

During a recent investigation it was necessary to find the natural logarithms of large numbers to at least seven places. The available tables were not extensive enough for the purpose. The use of common logarithms and the modulus was not satisfactory because a seven place table was the largest available. However, a ten-column computing machine was available and this suggested the preparation of a table to supplement the tables on page 257 of the Smithsonian Tables of Hyperbolic Functions.[1]

Table 1.

u	e^u
21	1 318 815 730.
20	485 165 195.
19	178 482 301.
18	65 659 969.1
17	24 154 952.8
16	8 886 110.52
15	3 269 017.37
14	1 202 604.28
13	442 413. 392
12	162 754. 791
11	59 874. 141 7
10	22 026. 465 8
9	8 103.083 93
8	2 980.957 99
7	1 096.633 16
6	403.428 793
5	148.413 159
4	54.598 150 0
3	20.085 536 9
2	7.389 056 10
1	2.718 281 83

* Published with permission of Geophysical Research Corporation.

[1] Smithsonian Mathematical Tables No. 1871. Becker and Van Orstrand, 1909.

The construction of the table is based on the expression

$$e^x = \sum_0^\infty \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$= \sum_0^\infty u_n \text{ where } u_0 = 1 \text{ and } u_n = \frac{x}{n} u_{n-1}.$$

While this converges for all values of x, it is needed only for $x < 1$ and the convergence is conveniently rapid. The results are shown in Table 2. Table 1 is taken from the Smithsonian Tables, for convenience.

TABLE 2.

u	e^u			u	e^u		
0.9	2.459	603	111	0.006	1.006	018	036
8	2.225	540	929	5	1.005	012	521
7	2.013	752	707	4	1.004	008	011
6	1.822	118	800	3	1.003	004	505
5	1.648	721	271	2	1.002	002	001
4	1.491	824	698	0.001	1.001	000	500
3	1.349	858	808	0.0009	1.000	900	405
2	1.221	402	758	8	1.000	800	320
0.1	1.105	170	918	7	1.000	700	245
0.09	1.094	174	284	6	1.000	600	180
8	1.083	287	068	5	1.000	500	125
7	1.072	508	181	4	1.000	400	080
6	1.061	836	547	3	1.000	300	045
5	1.051	271	096	2	1.000	200	020
4	1.040	810	774	0.0001	1.000	100	005
3	1.030	454	534	0.00009	1.000	090	004
2	1.020	201	340	8	1.000	080	003
0.1	1.010	050	167	7	1.000	070	002
0.009	1.009	040	622	6	1.000	060	002
8	1.008	032	086	5	1.000	050	001
0.007	1.007	024	557	0.00004	1.000	040	001

The use of these tables depends on the relation

$$\log n = \log y + \log \frac{n}{y}.$$

Let $y=e^u$ be the largest entry in the table of e^u for which $y \leqq n$ where $\log n = x$ is desired. Then if the tabular difference is Δu, we have $y=e^u \leqq n = e^x < e^{u+\Delta u}$. Hence $1 \leqq n/y < e^{\Delta u}$ and

$$0 \leqq \log \frac{n}{y} < \Delta u.$$

Thus $\log n$ is divided into two parts, one given in the table and the other between 0 and the tabular difference in the table. Even where no computing machine is available, the division is usually not more troublesome than the use of the modulus would be.

To illustrate the method, we proceed to find $\log_e 20\,000.500$. The calculations are shown in Table 3. The figures under u are the consecutive digits of the desired logarithm. The values under y are taken from Tables 1 and 2. Each value under n is obtained by dividing the value of n just above it by the value of y in its own line. As soon as a value of $n<1.00004$ is reached, the quantity $(n-1)$ represents the remaining digits of the desired logarithm. In practice the implied figures are omitted and Table 3 is replaced by Table 4, or replaced by machine calculations.

TABLE 3.

u	y			n		
9.	8103.	083	9	2.468	257	8
9	2.459	603	1	1.003	518	7
03	1.003	004	5	1.000	512	7
5	1.000	500	1	1.000	012	6

log 20 000.500=9.903 512 6

TABLE 4.

u	y			n		
9.	8 103.	083	9	2. 468	257	8
9	2. 459	603	1	3	518	7
03	3	004	5		512	7
5		500	1		12	6

log 20 000.500=9.903 512 6

This method is more convenient than the use of series, even for small numbers and the results are reliable to within a few units in the last figure. In the example, the error does not exceed three in the eighth figure.

HOUSTON, TEXAS,
JUNE 16, 1926.

Electric moments of the molecules of CH_4, CH_3Cl, CH_2Cl_2, $CHCl_3$, CCl_4. If the molecules of a gas have no intrinsic electric moment, but solely a moment proportional to the electric field, the dielectric constant should be independent of temperature and depend on the gas-density according to the Clausius-Mosotti formula; if however they have in addition a fixed intrinsic moment, a term of the form *const./T* should be added to the formula, the constant being calculable from the intrinsic moment. Measurements upon the dielectric constants of the gases cited in the heading showed that CH_4 and CCl_4 conform to the Clausius-Mosotti formula, while the others display a term *const./T* greatest for CH_3Cl, less for CH_2Cl_2 and least for $CHCl_3$. It may be inferred that symmetrical molecules lack intrinsic electric moment, while dissymmetry entails it. The intrinsic moments observed are of the order 10^{-18}.—[R. Sänger (Technische Hochschule, Zürich); Phys. ZS., *27*, pp. 556–563; 1926.]

KARL K. DARROW.

Effect of concentrated loads on the length of measuring tapes. In the standardization of base lines there is need for a precise formula for computing the change in length resulting from the addition of loads such as thermometers. A general formula has been proposed by W. D. Lambert, of the U. S. Coast and Geodetic Survey, and if this "be combined with the classical formula which expresses the difference in length of a tape when supported throughout and when it is supported at a number of equidistant points" there results the formula:

$$L_n' = L - \frac{L}{6}\left(\frac{\rho L}{2nW}\right)^2 - \frac{\rho}{2W^2}\sum_{k=1}^{k=N}\left\{\left[\left(\frac{L}{2n}\right)^2 - x_k^2\right]\left[w_k + \frac{n w_k^2}{\rho L}\right]\right\}$$

where

L_n' = length of tape supported in n loops with N concentrated loads

L = length of tape supported throughout at tension W

ρ = linear density of the tape

x_k = distance of the kth load, ω_k, from the lowest point of the loop in which it is placed.

The term involving the summation is shown to be small, amounting for example to 69 microns in a 50-meter tape supported at its ends and with a 50-gram load at its center, the tension on the tape being 15 kg.

A table is given showing the change in length of 50-meter invar tapes, of linear densities from 24.0 to 27.0 grams per meter, under a tension of 15 kg due to two thermometers weighing 45.0 g each placed 1 meter from the ends. With five supporting points this shortening amounts to about 65 microns.

An experimental test of the formula shows it to give correct results within a very few microns.—[Lewis V. Judson, Scientific papers of the Bureau of Standards, No. 534, August 24, 1926.]

F.K.R.

AN X-RAY SPECTROMETER WITH WHICH WAVE LENGTHS ARE READ DIRECTLY ON AN ORDINARY MICROMETER SCREW

By Warren W. Nicholas

In a recent number of this journal[1] the author described the principle of an x-ray spectrometer with which wave lengths could be read directly on a micrometer screw not especially calibrated for the purpose. Now that a spectrometer utilizing this principle has been in successful operation, it seems desirable to discuss the accuracy attainable with an apparatus of this kind, and also to describe several features of construction.

I. THEORY

The scheme for rotating the crystal table is shown in Fig. 1. The axis of the crystal table is perpendicular to the plane of the paper and passes through the point α. The table is rotated by means of the arm

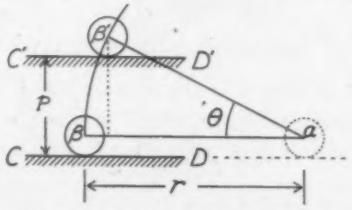

Fig. 1. *Scheme for rotating crystal table.*

$\alpha\beta$ of length r. On the end of the arm is fastened a ball whose center is at the point β. (The size of the ball is exaggerated in the drawing.) The ball bears against a plane surface CD which is perpendicular to the paper, and whose parallel displacement, p, from the position CD can be read on a micrometer screw. For the reference position, CD, the angle θ is zero, and the micrometer screw reads zero. For any position $C'D'$ the distance p equals the perpendicular distance from β' to $\alpha\beta$, so p/r is the sine of θ. Suppose the reflecting planes of the crystal are perpendicular to the paper, and the incident beam of x-rays is parallel to the plane of the paper. If the angle which the incident beam makes with the crystal is zero when θ is zero, then this angle is equal to θ for

[1] J.O.S.A. & R.S.I., *12*, p. 45; 1926.

any value of θ. Now it is well known that the wave length of the x-rays reflected from the crystal is equal to $\lambda = 2d \sin \theta$, for first order reflection, where d is the lattice constant of the crystal. Therefore p must be proportional to λ, and it is obvious that either the units of p or the distance r can be so chosen that p will indicate the x-ray wave length directly in angstroms or x-units (0.001 angstrom).

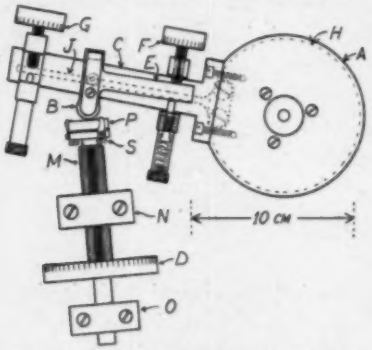

FIG. 2. *Plan of crystal table with micrometer adjustments (table H with its arm J of Fig. 3 in dotted lines).*

II. DESCRIPTION OF APPARATUS

The arrangement in Fig. 2 is the same as in Fig. 1. The crystal table A is rotated by means of the arm C to which is fastened a ball B. The ball bears against a piece of plate glass P, which is fastened directly

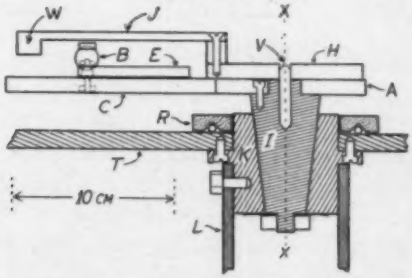

FIG. 3. *Vertical section of crystal table and supports. The parts of Fig. 2 which are included here are not cross-hatched.*

to the micrometer screw M. The orientation of P is adjustable by means of the three screws S. The micrometer screw is supported by the nut N and the bearing O. The position of the screw is read on the dial D. The distance of ball B from the axis of the crystal table is

adjustable by means of the arm E and the screw F. The crystal (not shown) is fastened to table H, which may be rotated with respect to table A by means of arm J and screw G. G makes contact with J at the point W shown in Fig. 3. The parts of Fig. 2 which are included in Fig. 3 are not cross-hatched. The crystal table turns on the conical spindle I whose axis is XX. The reference point V on this axis will be necessary in the adjustments. The bearing K is attached directly to the thick walled steel tube L which supports the entire apparatus. The ring R, which turns about K on ball bearings, supports the weight of the ionization chamber and counterbalance (not shown). To K is

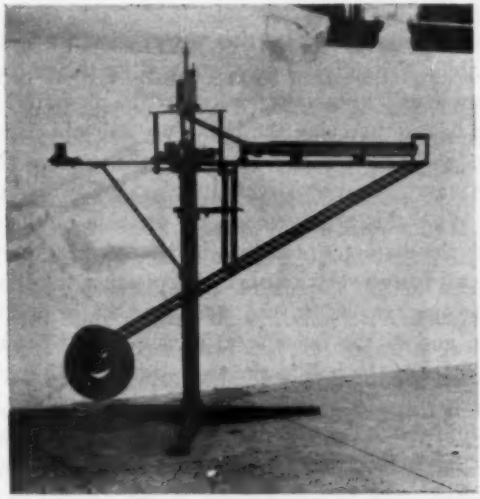

FIG. 4. *Spectrometer, electrometer, ionization chamber, and counterbalance.*

fastened the base plate T which was approximately 25 by 40 cm. To T are fastened the micrometer screw M and the slit system. To T are also bolted rods for supporting a platform on which is mounted the electrometer, directly above the crystal. This arrangement, as well as the slit system, and method for supporting the ionization chamber and its counterbalance, is shown in the photograph.

The ionization chamber was counterbalanced in order to avoid any changes in the level of the apparatus as the chamber was revolved. Although a Compton electrometer was used, no disturbing effects were found due to such changes of level. On account of the counterweight

and ball bearing support, the ionization chamber rotated very smoothly and easily.

In the present apparatus the pitch of the micrometer screw threads was 0.9998 mm and the circumference of the disc D was divided into 250 equal parts. Corresponding to 50 x.u. per turn of the screw (or 0.2 x.u. per division), and a lattice constant for the calcite 100 face at 18° of 3.02904Å, the center of the ball was set at a distance of 121.14 mm from the axis of rotation of the crystal table.

III. PRELIMINARY ADJUSTMENTS

An inspection of the geometry of the system shows that seven preliminary adjustments are necessary:

First, the ball must be set at the correct distance from the axis of the crystal table. A dividing engine was used for this purpose (the same engine had previously been used to measure the pitch of the micrometer screw). The base T carrying the entire crystal table was fastened to the traveling table of the engine in such a manner that the axis of the crystal table was vertical and accurately perpendicular to the bed of the engine. With this arrangement, on rotation of the crystal table through 180°, the diameter of the circle described by the center of the ball could be measured very accurately by setting the cross hair of the engine microscope tangent to the *outer* face of the ball for the first position, say, and to the *inner* face for the second position, the line joining the center of the ball in the two positions being parallel to the bed of the engine. The diameter once determined, it was easy to increase or diminish it by the desired amount by means of the micrometer F. After this was done, the diameter was again measured for a check. The result of this method is independent of the diameter of the ball, and is practically not dependent to any great extent on the position of the axis of the crystal table, this axis (i.e., point V) being used simply to line up the ball in positions 1 and 2. The ball was a good steel ball bearing $\frac{1}{2}''$ in diameter, and variations in its radius were probably well under 1/10,000 in. Errors in setting the ball, variations in the ball radius, and errors in measuring the screw threads may be grouped together and considered equivalent, in the present measurements, to a possible error of about 0.02 per cent in setting the ball at the correct distance from the axis of the crystal table.

Second, the outer face of the glass plate P on the micrometer screw (Fig. 2) must be adjusted to perpendicularity with the axis of the screw. For this purpose the glass face was lightly silvered, and a lens

of long focal length was mounted directly in front of it. A very small light source was then arranged near the axis of the screw, extended, so that light from the source passed through the lens, was reflected from the silvered surface, passed back through the lens, and was focussed on a wall 6.5 meters from the silvered face. On rotation of the screw, the spot on the wall moved in a circle. It was possible to adjust the glass so that the circle was about a millimeter in diameter. It was easy to show at the same time that the screw was not bent, by ascertaining that the circle described by the spot of light did not change appreciably in size as the screw traveled its entire length due to continuous rotation.

The third adjustment consists in setting the axis of the micrometer screw M perpendicular to the axis of the crystal table. The flange and the conical bearing surface of K had been turned without changing the position of K in the lathe chuck, so the surface of T was for all practical purposes perpendicular to the axis of the crystal table. It was therefore sufficient to adjust the axis of the micrometer screw parallel to the surface of T. This could be tested, with sufficient accuracy for the present work, by measuring the heights above the base T of one end of the screw for the extreme positions of the screw threads with respect to the nut. The heights were adjusted equal to within 0.2 mm.

The fourth adjustment was the setting of the micrometer screw to read zero when the line through the center of the ball and perpendicular to the axis of the crystal table was also perpendicular to the axis of the micrometer screw. This was done by first setting the screw at such a point, by means of a straight edge, that the plane determined by the glass face passed through the point V (Fig. 3), and then backing the screw a distance equal to the radius of the ball. This position was then the zero of the screw. The estimated possible error in this setting was 0.2 mm. The error due to a tilt of the glass face was negligible in this adjustment.

Fifth, the crystal should be adjusted so that the reflecting 100 planes are parallel to the axis of the crystal table. This was done to about 1/2000 radian by means of a square. It would be possible, especially when the crystal face is not very smooth, to make this adjustment satisfactorily by an x-ray method.

Sixth, the line joining the source of x-rays to the point of reflection of x-rays from the crystal should be perpendicular to the axis of the crystal table. This was done in the present case with sufficient accuracy by sighting along a ruler held parallel to the top of the crystal table and on a level with the center of the slit next to the crystal. In this

manner the center of the focal spot could easily be set to within 2 mm of the desired position. The focal spot was at a distance of about a meter from the crystal.

Seventh, the crystal was set so that its 100 planes were parallel to the axis of the slit system when the micrometer screw *M* read zero. This was done roughly by sighting, but the final adjustment was made by the use of the $K\alpha_1$ line of tungsten as a standard. For this purpose, as well as for the tests of section V, the x-ray beam was limited by two slits of equal width between the tube and the crystal. As Richtmyer has shown,[2] for this condition the line shapes are nearly triangular (see Fig. 5 for illustration). The adjustment was then made as follows: The micrometer screw was set for 208.71 x.u. (wave length of $WK\alpha_1$ from International Critical Tables), and the line was plotted as a function of the position of screw *G* mentioned above. Finally screw *G* was permanently set at the correct position to reflect the center of the line. It is not necessary, but it may be well, in these measurements, to have the slits lined with the axis of the crystal table, and the crystal face on this axis, in order that the x-rays reflect always from the same region of the crystal face.

IV. ERRORS[3] DUE TO IMPERFECT ADJUSTMENTS

In this discussion no errors will be considered which may have been due to strains in the various parts of the apparatus. In order to reduce these strains to a minimum the ball was not held against the glass face by means of a spring; a reading was approached from the short wave length side, the screw pushing the ball ahead of it until the desired position was reached.

Some types of error, as for instance the error in the ball setting *r*, are most readily expressed in percentage. The error in wave length measurement caused by this would be the same percentage only if the crystal angle were set correctly for zero wave length. Assuming the crystal angle to have been set perfectly for $WK\alpha_1$, these errors will have been reduced for wave lengths longer than $WK\alpha_1$, but increased for very short wave lengths. In the immediate vicinity of $WK\alpha_1$, the spectrometer will be, of course, very accurate. In the present work, due to the 0.02 per cent uncertainty of the ball setting, the possible wave length error at one Angstrom will approach 0.2 x.u.

[2] Richtmyer, Phys. Rev., *26*, pp. 724–35; 1925.

[3] The errors are rough approximations only. In each case, the maximum error, instead of the probable error, is estimated.

The error due to imperfect alignment of the glass plate perpendicular to the screw axis will be comparatively small when the ball is near the center of the glass face. Therefore it is advantageous in fastening the micrometer screw bearings to the base to select a position so as to minimize this error over as large a range as possible. In the present experiments the maximum distance from ball center to screw axis was about 6 mm, so the possible error in wave length reading, taking into consideration the tilt of the glass face given above, was about 0.03 x.u.

Suppose the axis of the micrometer screw makes an angle δ with the base T, which will be assumed perpendicular to the crystal table axis. Then when the screw traverses a length l along its axis, the ball will be pushed a distance $l/\cos\delta$. But when the threads are being measured, if the screw makes the same angle with the bed of the dividing engine, the length l will appear to be $l\cos\delta$. Thus the total error introduced by the inaccuracy will be $1-\cos^2\delta=\sin^2\delta$. δ was not greater than 0.004 of a radian in the present work, so the error must be under 0.002 per cent.

As to the fourth adjustment, suppose the line through the ball center and perpendicular to the crystal table axis makes an angle of $\pi/2\pm d$ with the micrometer screw axis when the screw reads zero. Then if $d>0$, the sine of the angle rotated by the crystal will not be p/r (see Fig. 1), but will be approximately $p/r\pm\theta^2 d/2$ where θ is comparatively small, and d and θ are measured in radians. The percentage error here is not constant over the entire range. In the present experiments, for instance, it amounted to about 0.01 per cent at one angstrom.

If the reflecting face of the crystal makes an angle a with the axis of rotation of the crystal table, the sine of the angle between the crystal face and the x-ray beam is reduced by a fraction approximately equal to $a^2/2$ where a is measured in radians. The same sort of error can be caused if the line joining the source of x-rays to the crystal at the point of reflection makes a finite angle b with the plane perpendicular to the crystal axis at the point of reflection of the rays. Here the error is, similarly, $b^2/2$. The combined errors from both these causes must be ess than 0.001 per cent in the present work.

V. EXPERIMENTAL RESULTS

A direct test of the characteristics of the spectrometer was made by measuring the WKα_1 line in several orders, the crystal angle having been set by the first order of this line. A consideration of all of the

above errors shows that the possible error in the measurement of the fifth order of $WK\alpha_1$ is about 0.4 x.u.

In Fig. 5 are shown the results of these measurements, and in Table 1 are given the numerical results together with the values for the apparent

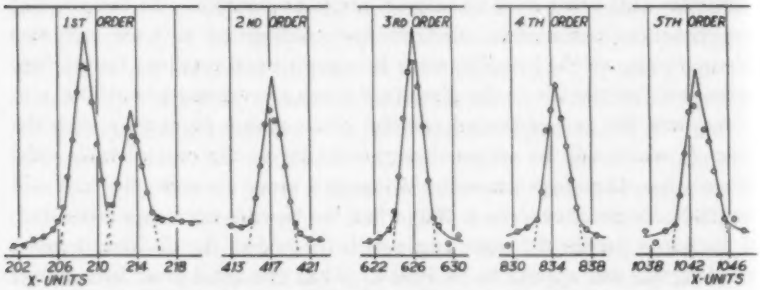

FIG. 5. *$WK\alpha_1$ in five orders, $WK\alpha_2$ first order. Intensity scales arbitrary. The abscissas represent "direct" readings from the micrometer screw M.*

wave lengths, obtained from the International Critical Tables. The slits used were 0.15 mm wide, and were 56 cm apart. The long ionization readings which were necessary on account of the small amount of energy getting through the slits made the wave length determinations uncertain to about 0.2 x.u.

TABLE 1.

Line	Measured wave length x.u.	Wave length from International Critical Tables[b]
$WK\alpha_1$ (1st order)	208.7[a]	208.71
$WK\alpha_2$ (1st order)	213.4	213.50
$WK\alpha_1$ (2nd order)	417.2	417.4
$WK\alpha_1$ (3rd order)	626.0	626.1
$WK\alpha_1$ (4th order)	834.4	834.8
$WK\alpha_1$ (5th order)	1042.7	1043.5

[a]Used for calibration.

[b]The correction in the various orders for refraction in the crystal is negligible here.

VI. DISCUSSION

It may be well to emphasize that for ordinary use the adjustments need not be made nearly so elaborately as here described. For instance, the micrometer screw threads and the distance from ball to crystal

table axis may be measured very satisfactorily with a large micrometer calipers. The other adjustments are comparatively simple, and are easily made. The fact that the spectrometer can be made in a shop not equipped for dividing circles is often a distinct advantage. Moreover in some types of work the direct reading characteristics are very desirable.

For precision wave length work it might be well to have the distance from ball to crystal table axis larger, say twice as large as in the present work. At least the glass face should be larger (the present one was 25 mm in diameter). But all the adjustments can be made with more precision than was done for the present tests. Especially lining the glass face with the crystal axis, and the setting of the ball at the correct distance from this axis, should be capable of much more accuracy. It is desirable, for this type of work, to set the crystal table angle independent of any standard wave length such as $WK\alpha$. This could be done by the use of a fairly thin crystal which had parallel reflecting faces on opposite sides.[4] A longer micrometer screw would then allow the ball to swing back to negative angles so that both sides of the crystal could be used. This system could be used for precision measurement of a particular line with decided advantage, especially if special adjustments were made for the purpose. For instance, the micrometer screw could be arranged so that, at the particular positions for reflection of the line, the ball would be near the center of the glass face (see section IV above). Moreover the error of adjustment four would appear twice and with different sign, so that this important error would be practically cancelled out. To test if the crystal faces were really parallel, the crystal could be turned end for end and the measurement repeated.

It is interesting to note that the position of the micrometer screw may be determined by interferometer methods in terms of light waves, and thus be independent of inaccuracies in the screw threads. This may be of value in determining small wave length differences. It is probably of academic interest only that the method here used makes the distance from ball center to crystal axis also available in terms of light waves.

[4] An alternative method for very thin crystals and very penetrating rays is one similar to that used by Rutherford and Andrade (Phil. Mag., *28*, pp. 263–73; 1914). The crystal is mounted crosswise of the beam, the planes used for reflection being approximately parallel to the short dimension of the crystal. The reflected x-rays, as well as the part of the beam not reflected, pass through the crystal. The advantage in this arrangement is that the same planes are used for reflection at both positive and negative angles.

It may be of greater practical importance that the readings may be made independent of temperature if, by a suitable choice of metals, the arm r (Fig. 1) is made to have an expansion coefficient greater than that of the screw and supports by an amount equal to the expansion coefficient of the crystal lattice.

The above work was done at Stanford University. I wish to thank Prof. D. L. Webster, Prof. F. K. Richtmyer, and Prof. P. A. Ross for valuable suggestions. I am indebted to Mr. S. W. Borrow and to Mr. B. Stuart, mechanicians, for construction of a large part of the apparatus.

CORNELL UNIVERSITY,
ITHACA, NEW YORK.
SEPT. 15, 1926.

Errata.—In abstract of paper on "Standard Artificial Sunlight for Colorimetric Purposes" by Priest, the following corrections should be made on page 480 of vol. 12; May, 1926:

(1) For "*Ives' summary of various data on special distribution,*" read: "*Ives' summary of various data on spectral distribution.*"

(2) The whole last line of the section of the table headed "*Smithsonian data on spectral distribution at Washington*" has been omitted. The missing line, to be inserted just above "*Bureau of Standards data*" should read:
"R.S.I. *6*, p. 563).....559.06 5605 .330 .335 .335 493 0.012"

(3) In the fourth line under "Bibliography," for "Phys. Rev. (2) II," read: "Phys. Rev. (2) *11*."

In abstract of paper on "The Minimum Perceptible Colorimetric Purity . . . ." by Priest and Brickwedde, pp. 306–307 of vol. 13; Sept., 1926, wherever "ρ" is printed, read "p."

STEREOSCOPIC PHOTOGRAPHY

By Susan B. Leiter

One use of the binocular microscope should be better known than it is. The difference in depth and the clearness of detail brought out by means of the binocular microscope can be photographically recorded for later reference. By means of the old fashioned stereoscope the truth of the picture can then be revealed as it is impossible to reveal it by the flat photograph. With an arrangement such as shown in Fig. 1, both pictures can be taken at the same time and the reversing and mounting of the print on suitable cards is a simple matter.

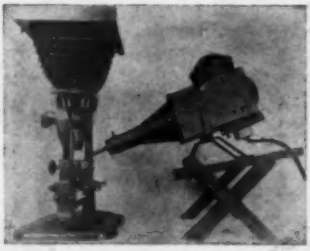

Fig. 1. *The Binocular Microscope Arranged for Stereophotography.*

Three illustrations are given on the following page—Fig. 2, the discharge from the end of a tungsten wire in a quartz tube; Fig. 3, a crystal loosened and ready to fall from a brittle material; and Fig. 4, a groove worn in an air pump valve.

Research Laboratory,
General Electric Company,
Schenectady, New York.

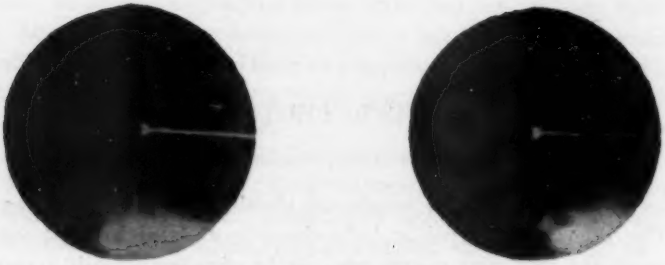

FIG. 2. *The discharge from the end of a tungsten wire in a quartz tube.*

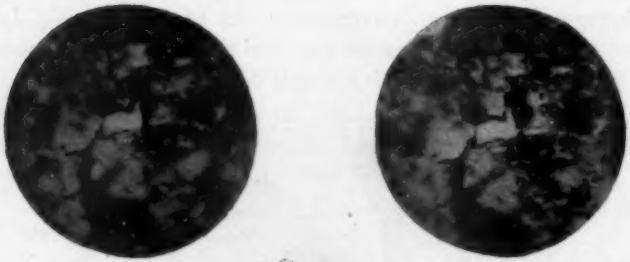

FIG. 3. *A crystal loosened and ready to fall from a brittle material.*

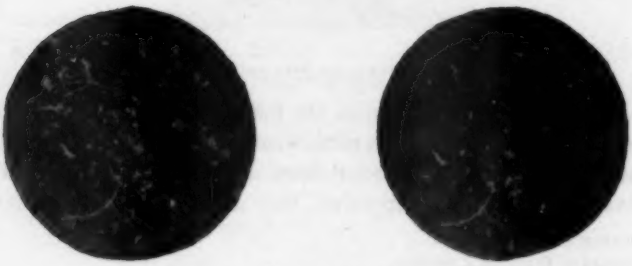

FIG. 4. *A groove worn in an air pump valve.*

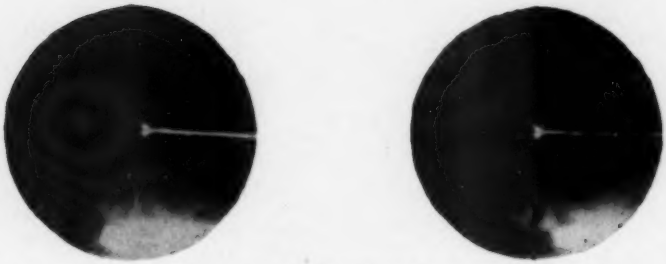

FIG. 2. *The discharge from the end of a tungsten wire in a quartz tube.*

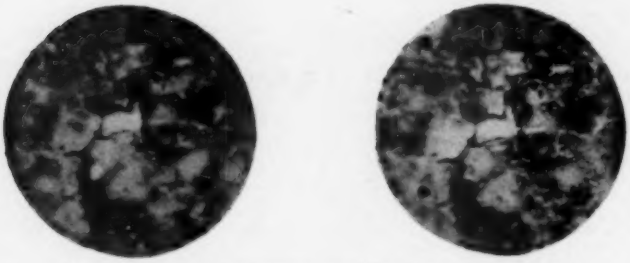

FIG. 3. *A crystal loosened and ready to fall from a brittle material.*

FIG. 4. *A groove worn in an air pump valve.*

BALLISTIC CHARACTERISTICS OF THE PHOTOELECTRIC CELL

By F. E. Null

INTRODUCTION

The problem of photoelectric emission is one of the most fundamental in physics, but our knowledge of the subject is limited to a few simple laws. It is known that the kinetic energy of the emitted electron is given by the relation, $\frac{1}{2}mv^2 = h(\nu - \nu_0)$, and that the number of electrons emitted is directly proportional to the intensity of the incident light, but our knowledge of the actual mechanism of emission is very meager. The work of O. M. Corbino and G. C. Trabacchi[1] indicated that a photoelectric current would not persist in a vacuum cell after the light was cut off, and Erich Marx and Karl Lichtenecker[2] found that the cutting of a weak incident light ray up into a very large number of small blocks of energy (of the order of 10^7 per sec) did not have any effect upon the photoelectric current.

In the work mentioned above, however, the exposures were made in very rapid succession. The effect of single isolated exposures was not determined, and as the mechanism of photoelectric emission is not known, the wave and quantum theories being fundamentally contradictory, and neither explaining the known field of phenomena, Dr. Kunz suggested a study of the effect of single pulse exposures, particularly as there was some demand for the photoelectric cell as a means of calibrating brief flashes of light. He, therefore, made one of his sensitive, potassium hydride, vacuum photoelectric cells, and this cell was used in all the following experiments.

DESCRIPTION OF EXPERIMENTAL METHODS FOR OBTAINING BALLISTIC CHARACTERISTICS

(a) *Condenser Storage Method.* It was desired to measure the quantity of electricity emitted by the photoelectric cell for single exposures of different length. The method consisted in exposing the photoelectric cell to a light beam for a given interval of time, and storing the electricity emitted by the cell in a large capacity condenser which at the end of the exposure was discharged through a ballistic galvanometer. The galvanometer could not be placed directly in series with the cell, for, (in most of the range of time exposures in which vacuum tube amplification was not necessary) the times of exposure were too short to allow the galvanometer to come to a steady deflection, and too long for true ballistic deflections. A condenser was therefore used to store the electricity passed by the cell during the time of exposure, and was then discharged through a ballistic galvanometer. (See Fig. 1.) A battery of 267 volts was used in

[1] O. M. Corbino and G. C. Trabacchi, Atti Della Reale Accademia dei Lincei, *5*, pp. 24, 908, 1173; 1915.

[2] Erich Marx and Karl Lichtenecker, Ann. der Physik, *41*, p. 124; 1913.

series with the photoelectric cell and condenser. The intervals of exposure were produced by means of a long rotating beam of light passing across a slit in front of the photoelectric cell.

The light system consisted of an incandescent lamp with straight filament as source, a lens system to form a rectangular beam of light, and a rotating mirror. A large condensing lens L_c was used to focus the light from the source upon a rectangular slit, S_1, the light from the slit being directed against the rotating mirror. A focusing lens, L_f, was placed between the slit and rotating mirror such that a distinct image of the slit passed across the receiving lens, L_r, in front of the photoelectric cell. Only the central portion of the focusing lens was used, in order to secure a clear image. The rotating mirror was driven by a low speed phonograph motor with a very large speed reduction by an intermediate pulley drive, at speeds of from 0.016 rpm to 1.1

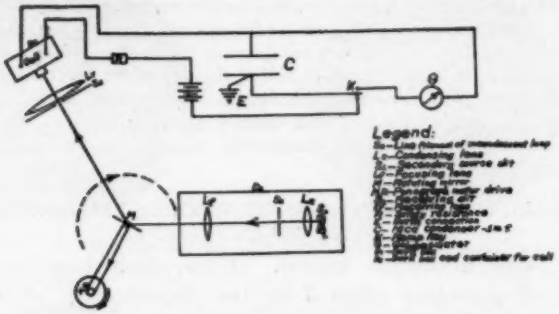

FIG. 1. *Condenser storage method.*

rpm for this method. The receiving lens in front of the photoelectric cell was used to condense a greater area of the image upon it. By placing a slit of the required width in front of the receiving lens, with its plane perpendicular to the axis of the beam, all of the light that falls upon the slit passes to the photoelectric cell. The slit can be made any required width, and its height can be made equal to that of the receiving lens, which is much greater than the window in the film of the photoelectric cell. The amount of light entering the cell was made as large as possible so as to produce readable galvanometer deflections for short time exposures.

All stray light was reduced to a minimum. The condenser was connected in series with the cell only at the instant the rotating light beam was in position to cross the slit, and as soon as the beam had passed was immediately discharged through the galvanometer.

(b) *Single Pulse Amplification Method.* The single pulse amplification method consisted in exposing the photoelectric cell to a rotating beam of light for very brief intervals of time, and in the use of a special, thermionic tube amplifier to increase the single pulses of current passed by the photoelectric cell, to quantities sufficient to be detected by a ballistic galvanometer.

The same light system was used as for the condenser storage method except that the beam of light was rotated at a much higher velocity, and the times of exposure were relatively very short. A battery of several hundred volts was put in series, with the photoelectric cell, a safety resistance of 100 000 ohms, and the grid leak of the amplifier system, as shown in Fig. 2.

The amplifier system was designed for single pulse amplification. A condenser could not be used to store the quantity of electricity emitted by the photoelectric cell (as in the method of Condenser Discharge), and to discharge it through the grid leak of the amplifier circuit, for the amplified current is not in this case proportional to the quantity of electricity passed through the grid leak, but is a function of the time of condenser discharge. The discharge curve for the condenser could not be conveniently obtained, as a commercial five megohm grid leak was used, and its resistance was not accurately known. The chief problem in the design of the amplifier was to differentiate between the d.c. plate current, and the amplified pulse current which was of the order of 10^{-4} times as large. In order to extend the amplified characteristic to meet that of the condenser storage method it was necessary to use light exposures as long as 0.1 sec. This requirement prevented the use of a choke coil to shunt off the d.c. current, for a choke coil would have to be very bulky and costly if it is not to produce distortion of the wave form at frequences as low as 10/sec.

The only available method therefore for separating the d.c. and pulsating plate currents, was by the use of a bucking battery and shunt that would produce a current flowing through the galvanometer in such a direction as exactly to compensate for the d.c. plate current, as shown in Fig. 2. But the use of the bucking battery and shunt is very difficult on account of the extreme difference in size of the d.c. and pulsating currents; i. e., a difference of 1/100 000 of a volt in either the filament or plate voltages would cause a change of at least 10 per cent in the measured pulsating current. To prevent effectively fluctuations in the d.c. plate current of the order of 1 part in 100 000,

requires great care. There are two serious disturbing factors: (1) change of resistance and of battery emf with temperature, and (2) fluctuation in voltage due to local action and change in ion concentration.

Local action is always present to some extent in lead storage cells; bubbles rose to the surface at frequent intervals and were supposed capable of changing the voltage of the cells by 1 part in 100 000. It was found necessary to use a dry cell filament battery (with a large number of cells in parallel to reduce polarization to a minimum) since the local action of the dry cell is negligible. All batteries were thermally insulated with cotton waste and asbestos coverings. An "amperite" (thermally insulated) was used as the filament resistance, since it has the property of a large change in resistance for a relatively small change in current, and was found to give as good results as a

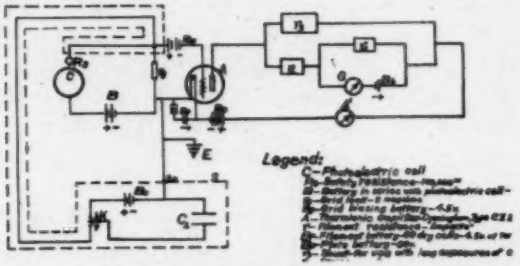

FIG. 2. *Single pulse amplification circuit.*

maganin wire resistance in an oil bath. It should be mentioned in addition that all connections were either soldered or made by screw binding posts, and that in order to eliminate external electrostatic and inductive influences, the wires to the grid were suspended by sulphur plugs through the centers of large glass tubes covered with an earthed iron wire screen. It was not found possible with the apparatus available to eliminate all drift from the galvanometer deflection, but was found to be sufficiently steady, for short periods of time, to allow ballistic readings to be taken.

It was found that the sensitivity of the amplifying system was variable, and that in order to obtain consistent results, it was necessary to measure the sensitivity of the amplifier immediately after taking each reading produced by exposure of the photoelectric cell. The reference used as a calibrating indicator was a well insulated air condenser that could be charged to a maximum of three volts and

discharged through the grid leak of the amplifying system without exceeding the grid bias on the three electrode tube. Each deflection produced by the cell was divided by that produced by the reference at the same time, and hence, the deflection of the photoelectric cell per unit sensitivity of the amplifying system could be obtained. For longer times of exposure of the photoelectric cell it was necessary to use the shunt r_3, Fig. 2, in the plate circuit in order to keep the galvanometer deflections on the scale, but for the shortest exposures deflections of only approximately 10 mm were obtained. Exposure was obtained at the desired instant by actuating a lever that allowed a shield to drop from in front of the photoelectric cell.

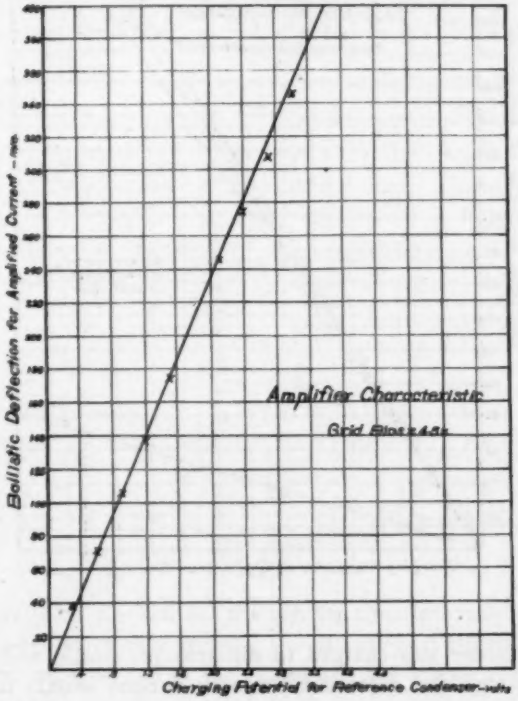
Ballistic Deflection for Amplified Current – mm.
Amplifier Characteristic
Grid Bias = 4.5 v.
Charging Potential for Reference Condenser – volts

FIG. 3.

RESULTS

Single Pulse Amplifier Characteristic

This characteristic was taken by charging the reference condenser to different voltages, and comparing the deflections obtained for the

amplified currents. The straight line portion of the tube characteristic was used. The points plotted in Fig. 3 are each the average of ten readings.

Shunt Characteristics of Ballistic Galvanometer

In order to be sure (for the resistances used) that the ballistic galvanometer deflection was directly proportional to the quantity of electricity passed through the shunt and galvanometer in parallel,

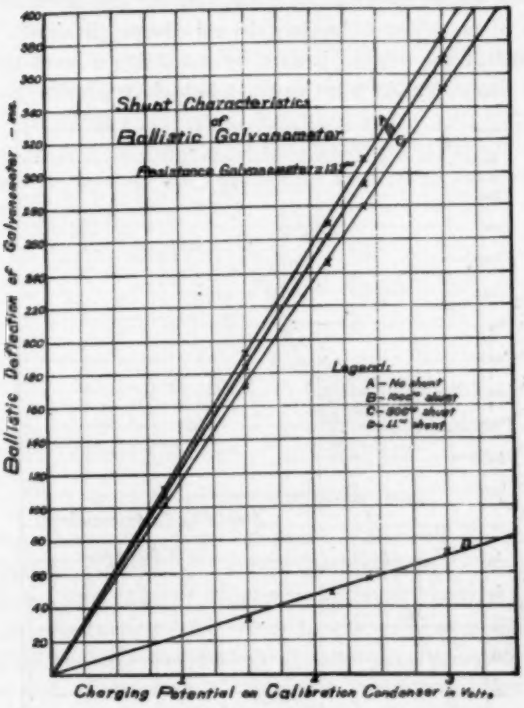

FIG. 4.

a 1 mf condenser was charged to different potentials, and then discharged through the galvanometer with various shunts in parallel. The results are plotted in Fig. 4.

Ballistic Characteristics of the Photoelectric Cell

Beam No. 1. Constants of Set-Up

Receiving slit width = rotating beam width = 18.5 mm.

Distance from mirror center to receiving slit = 381 cm.

Since the beam width is exactly equal to that of the slit, the time of exposure is made up of two portions, (1) the time required for the beam to cover the slit, (2) the time required for the beam to uncover the slit. The sum of the energy delivered to the cell in periods (1) and (2) is equivalent to that delivered by the entire beam for a period of time equal to one-half of that required for the beam to pass the slit, or, T(the time of exposure in seconds) is,

$$T = \frac{1.85 \times 60}{2 \times 381 \times 2} \times \frac{1}{r.p.m} \text{ seconds.}$$

The relation is obtained between the quantity of electricity emitted by the photoelectric cell and the length of the time of exposure. The condenser storage method was used for the longer times of exposure,

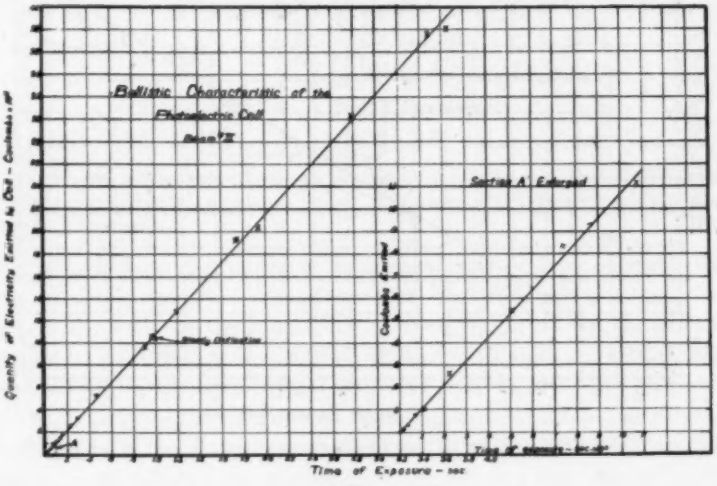

FIG. 5.

and the amplifier method for the shorter time intervals of exposure. In the condenser storage method the quantity of electricity emitted by the photoelectric cell at each exposure was stored upon a one mf condenser which was then discharged through a ballistic galvanometer, while in the amplifier system the quantity of electricity emitted by the cell was discharged through the grid leak of the amplifier circuit. A straight line relation was obtained by both methods, and since the range of time exposures covered by the two methods overlapped, the

slope of the characteristic obtained by the two methods was identical, and the absolute values by the amplifier method could be obtained from the slope of the characteristic. The data are plotted in Fig. 5.

BALLISTIC INTENSITY CHARACTERISTIC AMPLIFIER METHOD

For this characteristic the length of exposure was kept constant, and the intensity varied, thus, giving a relation between the quantity of electricity passed by the photoelectric cell for different intensities. The most convenient method for varying the intensity of the light was by placing two large nicol prisms in the path of the beam before it struck the rotating mirror. The position of the prism adjacent

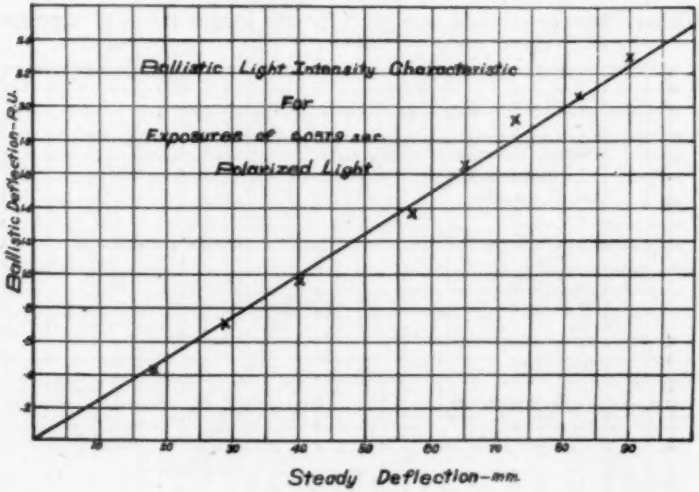

FIG. 6.

to the mirror was kept constant so that the plane of polarization of the emerging beam always had the same angle of incidence with the photoelectric cell. Then by rotation of the other nicol prism, the intensity could be reduced to any required amount. Several fine marks were made on the mounting of the nicol prism, and the quantity of electricity emitted by the photoelectric cell for both steady and pulse exposures was obtained for each mark, and their ratio gave the slope of the ballistic deflection-intensity characteristic for each intensity. The characteristic is plotted in Fig. 6.

CALIBRATION OF CAMERA SHUTTER

We have shown by previous experimental work that the quantity of electricity emitted by the photoelectric cell per unit time is (for incident light of constant wave length) directly proportional to the light energy received by the cell per unit time regardless of the distribution of this incident energy with respect to time. The photoelectric cell is, therefore, a very convenient device for calibrating camera shutters or light interrupters of any type, being first used for that purpose by Richtmyer.[3] All that is required is to place the shutter between the photoelectric cell and the light source, Fig. 7, and to measure the quantity of electricity emitted by the cell for different settings of the shutter. If for a stop opening of a given size, and time setting of the shutter, we measure the steady deflection produced by the electricity emitted by the photoelectric cell, and divide this by

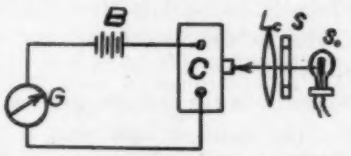

FIG. 7. *Shutter calibration by the photoelectric cell.* S_0, *light source (incandescent lamp); S, shutter to be calibrated;* L_c, *condensing lens; C, photoelectric cell; B, battery of several hundred volts; G. galvanometer.*

the ballistic deflection produced by a given time setting of the shutter, and multiply by F(coulombs per sec./mm)/G(coulombs/mm), we obtain the ratio of the quantities of energy emitted by the cell for exposure (to a beam of constant intensity) for one second, to that emitted during the opening and closing of the shutter. The reciprocal of this ratio gives the time of the shutter operation, or the time interval during which the shutter would be open if it had 100 per cent efficiency, i. e., if the time required for the opening and closing of the shutter were zero. If the area of the stop opening is made small relative to that of the shutter opening, the time phases of opening and closing will be practically zero, and the shutter will have nearly 100 per cent efficiency for this stop opening. If now a number of readings are taken for different stop openings, but for the same time setting of the shutter, a curve may be obtained between the galvanometer deflection and the area of the stop opening. If the second stop opening is made but slightly larger than the first, the shutter will have approximately

[3] F. K. Richtmyer, Phys. Rev., *30*, pp. 394–6; 1910.

100 per cent efficiency for these openings and a straight line may be drawn through these points and the origin. This is the line of 100 per cent efficiency, and the ratio of an ordinate on the curve to an ordinate on the line gives the efficiency of the shutter for the given stop opening.

Thus, the efficiency can be obtained for any stop opening or segment of stop opening, but this does not enable us to obtain the opening and closing curves (stop opening area plotted against time) for although we can determine the total time required for the shutter to open and close over any segment of the stop opening, we have no means of dividing this time between the opening and closing phases.

For many practical problems however, we are interested in the total amount of light energy received, and not in the rate of light reception at a particular instant in the interval of exposure, and the photoelectric cell is particularly adapted to that type of calibration. It gives a convenient check on the performance and working order of a shutter at any time.

It is necessary, however, in the experimental manipulation to take certain precautions. The beam of light that is incident upon the photoelectric cell should always be brought to a focus, and allowed to diverge before striking the sensitive layer of the photoelectric cell which otherwise might become overheated, and change its sensitivity. Also, the photoelectric cell should never at any time be exposed for steady deflection to light of the same intensity as is used for ballistic deflection. For obtaining the steady deflection, the light source should either be removed to a distance, or a small auxiliary stop opening placed in front of the shutter. The resulting steady deflection is multiplied by the ratio of the stop area used for the ballistic deflection to the area of the auxiliary stop used for the steady deflection. The light source should be on the normal to the shutter plane, as otherwise distortion of the image will occur, and the position of the diffuse image on the photoelectric layer will change with the diameter of the stop opening. To avoid heating of the shutter, the light source should be turned on only while actual exposures are made, and it is better to keep the light source as far away from the shutter as will allow sufficient ballistic deflections to be obtained. If a "Pointolite" is available, the light source can be kept at a distance from the shutter, and a uniform beam of sufficient intensity can be directed against the shutter by means of lenses.

It was desired to calibrate a shutter, and to compare the times of operation obtained by the photoelectric method with those obtained by the photographic method used by the company that manufactured the shutter. The photographic method of calibration consisted in

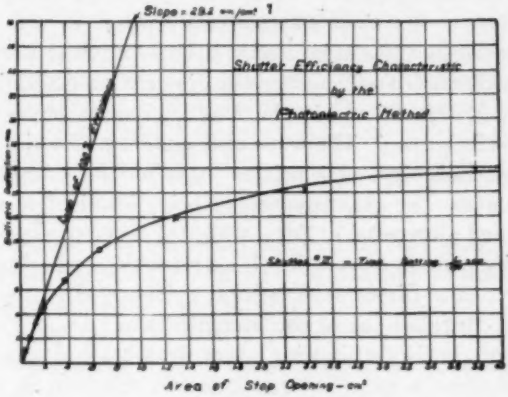

FIG. 8.

taking one photograph of the shutter every 1/1000 second. The approximate length of the time of shutter operation is obtained by counting the number of pictures of the shutter for a given time setting. If the shutter starts to open, or completely closes between the time

TABLE 1. *Determination of absolute time* of shutter operation*

Shutter mark	Timing sec.	Ballistic deflection mm	Steady deflection mm	Actual time of operation	
				by photoelectric method	by photographic method
1/10	0.100	19.0	305.6	0.109	
1/25	0.040	7.28	305.6	0.0416	0.410
1/50	0.020	3.32	305.6	0.0190	0.019
1/100	0.0100	1.46	305.6	0.00337	0.0090
1/150	0.0066	1.46	305.6	0.00837	0.0090

*The efficiency characteristic is plotted in Fig. 8.

of two successive exposures of the calibrating camera (1/1000 sec.), an appreciable error will be found for short times of exposure (as few pictures are then taken). The following data indicates that the photoelectric and photographic methods of calibration of the time of shutter

operation agree closely for the longer times of exposure, and approximately for the shorter times of exposure.

PROPOSED EXTENSION OF WORK

Method of Detecting Quanta by Means of Change in Frequency of Amplified Photoelectric Currents

The photoelectric cell can be used in a simple method for differentiating between the wave and quantum theories, i. e., to determine whether light energy is transmitted in the form of quanta or as continuous electromagnetic waves. The wave and quantum theories are fundamentally contradictory, and neither explains the complete field of light phenomena, the quantum theory being required to explain the production and absorption of light, while the wave theory is necessary to explain its periodic properties, i. e., refraction, polarization, interference, etc. Any direct experimental evidence as to the nature of light should, therefore, be of considerable value.

The method consists in allowing a very weak beam of monochromatic light to fall upon a gas filled photoelectric cell, and in magnifying the photoelectric current by means of an ultra sensitive amplifier which has a wave meter coupled to its output circuit, so that the frequency of the photoelectric current can be determined. This frequency is of course determined by the frequency of exposure of the photoelectric cell to light energy. By the use of a series of pin hole reductions, the measured intensity of a beam of monochromatic light can be reduced to any desired calculated value. Then if we assume the quantum theory, we can readily calculate the average number of quanta incident upon the photoelectric cell per second. Also, if we assume that quanta travel from the source in a regular, accurately spaced succession, as seems required by the well established periodic properties of light (refraction, polarization, interference, etc.), the frequency of exposure of the photoelectric cell would equal the number of incident quanta per second, say 100 000, and this frequency would be indicated by the wave meter. Moreover, if the intensity of the beam was reduced to one-half its former value, the number of quanta incident upon the photoelectric cell would be reduced to 50 000 per second, and hence, the frequency of exposure of the photoelectric cell and of the amplified photoelectric current would be reduced to 50 000 pulses per second. If on the other hand, quanta travel in clusters or in random succession, their group frequency would be given by the wave meter. By the wave theory, however, the incident

light energy would consist of electromagnetic waves of such very high frequency that no current would flow in the wave meter circuit. Thus, a frequency in the wave meter circuit that changed with the intensity of the incident beam would be positive evidence of quanta. The success of the method depends upon an ultra sensitive amplifier. By using a gas filled photoelectric cell with a voltage somewhat less than its flashing point in the dark an amplification of several thousand fold should be obtained inside the cell from ionization by collision, and if the output of the cell is impressed upon a sensitive three electrode amplifier, the frequency of exposure of the photoelectric cell should be capable of detection.

CONCLUSIONS

1. The quantity of electricity emitted by the photoelectric cell for single flash exposures is, for light of constant intensity, directly proportional to the length of exposure.

2. The quantity of electricity passed by the photoelectric cell for single flash exposures of constant length, but consisting of plane polarized light of different intensities, is directly proportional to the intensity of the light.

3. A single pulse amplifier can be obtained by the use of a bucking battery, that will give linear amplification for pulses of current whose length varies over a range of from one tenth of a second to approximately six ten-thousandths of a second.

4. The photoelectric method of camera shutter calibration is convenient and accurate.

5. The photoelectric cell provides a means for distinguishing between the wave and quantum theories.

I am indebted to Professors Carman, Schulz, Knipp, Watson and Williams for coöperation in lending apparatus, and wish to express my appreciation of the assistance of Dr. Jacob Kunz who personally directed the work.

UNIVERSITY OF ILLINOIS,
URBANA, ILLINOIS.

Transfer of energy from excited Hg atoms to Zn atoms. A mixture of Zn vapor with Hg vapor, irradiated by light from a mercury arc including the line $1S-2P$ and therefore able to put the Hg atom into the $2P$ state differing in energy from the normal state by 6.7 equivalent volts, emits lines of Zn having excitation potentials as high as 8.48 volts. When the line $1S-2P$ is cut out of the irradiating light, leaving only the line $1S-2p_2$ able to put the Hg atom into an excited state having 4.9 equivalent volts more than the normal state, the lines of Zn requiring high excitation potentials are very much reduced in intensity, while the resonance line of Zn (excitation potential 4.01 volts) is almost unchanged. The same irradiation of Zn vapor not mixed with mercury excited no lines whatever.—[J. G. Winans (Wisconsin); Nature, *118*, pp. 46–47; 1926.]

KARL K. DARROW

Anregung von Quantensprüngen durch Stösse. By J. Franck and P. Jordan; 312 pages; Julius Springer, Berlin; 21 marks.

The publication of this book is an important event in the world of science. Probably half of the current research work in physics and physical chemistry is related to the general subject of excitation by collision, and Prof. Franck and his students are responsible both for the pioneer work in the field as well as for many of the important subsequent developments. The book starts with a chapter on scattering of electrons by gases. Two long chapters give the methods of measuring critical potentials, the results obtained in monatomic vapors and gases and their interpretation in terms of spectrum series. This field has been covered in several recent books but the present presentation has the advantage of giving all spectrum relations in terms of the new multiplet theory and series notation.

Progress in the subject of molecular excitation as related to band spectra has been so rapid that the chapter on this will soon be out of date though foot notes added to the proof indicate the trend of recent developments. One of the most important chapters is that on the exchange of excitation energy, covering the subjects of duration of excited states, collisions of the second kind between electrons and atoms and between different kinds of atoms, etc. There are also chapters on the probability of excitation by electron collision, on excitation by temperature and on photochemical processes. The authors have succeeded in covering a comprehensive subject by adhering to a concise style. Theoretical conclusions are in general stated without any explanations as to their derivation but ample references are included which will enable the student to readily fill in these gaps by outside reading.

F. L. MOHLER

A NEW TYPE OF ASPIRATOR OF HIGH EFFICIENCY

By E. L. Harrington

In connection with some work done in developing a mercury multi-stage diffusion pump capable of operating against a sufficiently high fore-pressure to enable an aspirator to serve as a fore pump, considerable difficulty was experienced in getting an aspirator that could be depended upon in this rather exacting service. Certain theoretical considerations and an extended experimental study of aspirator performances led to an interesting study of the relative importance of the Bernoulli principle and the viscosity of air in aspirator action, and finally to the design of a very simple, yet highly efficient aspirator.

The aspirator, or filter pump, has so long been the stock text book illustration of an application of Bernoulli's principle that the latter is generally supposed to furnish the complete explanation. Of the various types on the market perhaps the two most widely known and generally used aspirators were evidently designed to utilize only this principle. The algebraic expressions of Bernoulli's theorem vary somewhat, but the following may be taken as typical:

$$P + g\rho h + \tfrac{1}{2}\rho V^2 = \text{const.}$$

The three terms given are obtained by considering the energy of[1] the liquid in the stream due respectively to its pressure, height and velocity.

Without going into great detail as to the application of this principle to the aspirator, it may be stated that in an aspirator the water has its highest velocity at the point of greatest contraction of the jet which is very near the end of the nozzle. Assuming an inappreciable change in the height, that is ignoring the second term, it is seen that this point must be the point of lowest pressure within the stream. Similarly, the lowest velocity and highest pressure are at the tail end. Since this latter pressure is that of the atmosphere, it follows that the pressure at the nozzle must be less than atmospheric by an amount which, in accordance with considerations based on Bernoulli's principle, depends upon the velocities at the two points. But it must depend also upon certain losses suffered by the stream which are not represented in the above expression, and which in some cases are negligible. In the case

[1] See Duff, Textbook of Physics, 5th Edition, p. 135; or Stewart, Physics, p. 168.

of the aspirator these losses are of such great magnitude that the simple expression for the theorem is quite useless.

Among these energy losses may be mentioned: (1) The loss due to tube friction between the nozzle and the end of the exit tube. (2) The loss due to changes in diameter at the ends of the exit tube. (3) The losses due to baffles, bends, or zig-zag tubes usually employed in commercial aspirators, very serious losses in some types. (4) The loss due to turbulence resulting from the admixed air, and varying with the amount of air passing.

For certain types of these losses treatises on hydraulics[2] give empirical formulas, but as such formulas apply only to ordinary piping they cannot be used in the present case. In fact it does not seem possible to calculate from theoretical considerations the exact magnitude of these losses, though it is certain that they are large. Whatever their value their effect is to place a lower limit on the value of the pressure obtainable by the aspirator. The only approach to the question seems to be through an experimental study of the aspirator itself.

Although in a well designed aspirator this limit might be well below the general limit of aspirators, i.e., the vapor pressure of water, yet it is true that a number of aspirators have been tested which actually do not reach even approximately this limit. By assuming the validity of Bernoulli's principle as the explanation of aspirator action one might explain such cases by supposing that the lower limit fixed by the above equation lies above the vapor tension of water, or if not, so near to it that the operating pressure difference becomes too small to be effective at the lower pressures.

Even in the case of the best pump of this type—granting that there is an effective differential pressure—just how can a lateral pressure on the jet cause the removal of the air? Consider the point at which the aspirator jet is first exposed to the air to be exhausted. The heart of the jet has high velocity and low pressure exactly in accordance with Bernoulli's principle. But the air is not in contact with this heart, and therefore cannot move with it. The surface of the jet on the contrary, has at the instant of leaving the nozzle, practically zero velocity, and though the air presses against it the air cannot move faster than this surface moves, and if only a short length of the jet is exposed to the air (as appears to be the usual case) the surface will not attain the mean jet velocity until it is too late for effective use. Even

[2] See e.g., Slocum, Elements of Hydraulics, p. 72 ff.

if it did, this mean velocity is only $.577V$,[3] where V is the original velocity of the heart of the jet. Granting the contact layer were removed, this would be unimportant unless the adjacent layers of air were carried along. The pressure gradient could not accomplish this since the pressure is lowest at the nozzle rather than at the outlet from the air chamber. The only way these air layers can be removed is through the agency of the viscosity of air, and by becoming entrained in the jet itself and mechanically ejected. Since the operation of these two factors is independent of the Bernoulli principle the author was led, as others apparently have been, not only to question the adequacy of the latter to explain aspirator action, but even to doubt its importance. Baker[4] definitely holds this point of view, and Friedrichs,[5] in his discussion of the theory of filter pumps does not explicitly mention Bernoulli's principle. In view of these considerations and certain experimental work it seemed worth while to attempt the design of an aspirator that might more fully utilize these viscosity and mechanical factors, in the hope that a more efficient aspirator might be developed.

This raises the question as to what would be considered an efficient pump. One might define the efficiency as the ratio of the work done in removing a certain quantity of air to the energy expended by the jet, but the quantities involved are too intangible. To the writer it appeared that the two factors of practical interest are the speed of exhaustion, and the water consumption. It was therefore decided to adopt arbitrarily as the index of efficiency the product of the speed in cubic centimeters per second by the number of seconds required to use one liter of water. Certainly the rating of an aspirator should vary directly with the speed, and of two pumps showing the same speeds the one requiring the greater time to consume say a liter of water would be considered the more efficient. The product, then, of these two factors should give a rational and convenient index of efficiency.

Assuming such a basis, then, it would appear that to be efficient an aspirator designed to utilize the viscosity principle should possess the following characteristics: (1) A high velocity jet, since the viscosity drag is proportional to the velocity gradient, and therefore to the

[3] This value is obtained by assuming that at the point where the jet has attained uniform velocity throughout its cross section the jet energy is the same as at the nozzle where the velocity variation over the jet's cross section is large, but known. See Poynting and Thomson, Properties of Matter, p. 208.

[4] Baker, Phys. Rev., *14*, p. 228; 1919.

[5] Friedrichs, Zeits. Techn. Physik, *6*, pp. 361–65; 1925.

velocity itself. (2) A large exposure of the jet surface to the air to be exhausted, as the effective viscosity force is proportional to the area of contact. (3) An exit tube designed to utilize fully the results of viscosity action and to insure stability of action at all pressure differences. (4) A relatively small water consumption.

These requirements, and the means employed to meet them, will be discussed in order:

1. *Jet Velocity.* The water pressure of the system to which the aspirator is attached sets a practical limit to the jet velocity that can be obtained. Systems having abnormally low pressures due either to the system itself or to the relative altitude of the laboratory will hardly be able to make satisfactory use of aspirators. Certain advantages, which are discussed in treatises on hydraulics, can be obtained by adopting efficient nozzle designs. Baker (l.c.) has shown that the nozzle should be made no longer than is necessary to give form and direction to the jet if the very considerable energy losses due to the viscosity of water are to be avoided.

2. *Jet Exposure to Air.* The steps taken to insure a large jet exposure proved to be the most important development of the present study. So long as one uses only nozzles of circular cross-sections the surface exposures increase with the first power of the radius, while the water consumption goes up with the fourth power, approximately. Hence the nozzle of large diameter would be bad design. Since that part of the jet which passes the greatest quantity of water is its heart and this heart is not in contact with the air, the obvious remedy is to omit it, virtually, by providing some means of obtaining an air exposure in the interior of the jet. The first scheme tried for accomplishing this is shown in Fig. 1. In this aspirator a small inner tube, *I*, leads air from the air chamber to the heart of the jet, which is thereby made annular in cross-section. Not only does this step greatly increase the jet exposure for a given water consumption but it produces a further increase in efficiency through the fact that the air in the interior of the jet is bound to become entrained and ejected mechanically. Since this annular jet may be made as thin-walled as is desired, it is possible to obtain any desired ratio between the surface exposure and the jet cross-section.

Glass aspirators constructed on this plan proved highly efficient but the design is not so well suited for construction in metal. Various methods not involving the use of the inner tube were tried for increasing the ratio of the perimeter of the jet to its cross-sectional area, which

is the essential thing. Some of these made use of obstructions in the nozzle to divide the jet into parts in order to insure greater exposure to and trapping of the air. But the scheme finally adopted is at the same time the simplest and the most satisfactory. It makes use of a group of small jets sufficiently separated to admit air between them into the heart of the composite jet in the manner shown in Fig. 2. Two advantages of this plan are the simplicity and the absolute control afforded over the form and direction of the jet parts. Incidentally the length of the nozzles need be no greater than is necessary to secure this

FIG. 1

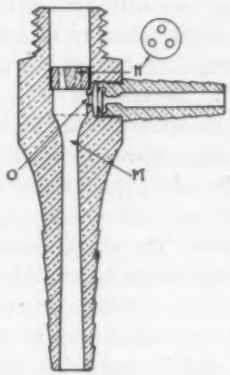

FIG. 2

FIGS. 1 and 2. *Longitudinal sections showing the construction of the new type aspirators. Fig. 1 shows a glass aspirator, inner tube form, Fig. 2 a metal aspirator, multi-jet form.*

control, so that the nozzle losses above mentioned may be kept at a minimum. Three jets were found to be sufficient for the purpose.

Another method of increasing the jet exposure is by the exposure of a longer portion of it. This has certain advantages. It is simpler than giving the nozzle a too complicated cross-section, and the viscosity drag on the air is actually proportional to a somewhat higher power than the first power of the length exposed since both the perimeter of the jet and the roughness of the surface increase with the distance from the nozzle. Evidence of these was obtained by means of an

aspirator so constructed that the distance between *N* and *M* could be varied considerably. The efficiency increases rapidly with the first few millimeters, passes through a maximum, and then rapidly decreases. Further evidence was obtained by making a modification in a commercial aspirator having but a very short jet exposure. After making an efficiency test the nozzle of the aspirator was cut off enough to expose an additional 6 mm of the jet to the air. This one change, though slight, increased the efficiency by thirty-one per cent. The optimum distance between the nozzle and the throat depends upon the kind of jet produced by the nozzle and the shape of the throat at *M*. If the jet is allowed to spread too much before reaching *M* a splash is produced there which interferes with the entrance of the air current into the throat. About 1 cm is found to be a satisfactory distance. In order to utilize the full length of the jet the air inlet, *O*, should be as close to the nozzle disk, *N*, as possible.

The reality of the drag due to the viscosity of the air can be demonstrated in a striking manner by holding a small flame close to a water jet of high velocity. It is this "wind" that an aspirator of the new type is designed to utilize.

3. *The Exit Tube.* The throat form and the nozzle are so mutually dependent that they must be considered together. The throat of the exit tube should have a cross-section considerably larger than that of the nozzle but yet not large enough to involve any risk of having the impact force of the jet exceeded by the back pressure due to the atmosphere at the lowest water pressure likely to be encountered in ordinary service. The best ratio for these cross-sections will depend upon the water pressure in a manner which is treated by Friedrichs (l.c.). Below the throat the cross-section of the exit tube should increase at a rate which depends upon the nature of the jet and upon energy considerations mentioned above in connection with Bernoulli's principle. The essential thing is to make it possible for the jet to fill the tube completely at every point, so that the increase in cross-section must be just enough to take care of the velocity losses. If it increases more rapidly there will be return passages for the air, while if less rapidly undue head losses will result. The total air-water volume delivered by the exit tube per second depends upon the state of exhaustion and is least when the exhaustion is practically completed. It is thus impossible to have a tube that will function best at all stages.

In cases where the exit tube is so large or the jet has such form that it can shoot through the exit tube without completely closing the latter

the aspirator will not function unless one starts a "splash" in the tube by partially covering the lower end, thereby reducing the effective diameter. Such a splash will seal the tube and permit the jet to do work against the air pressure, by which the velocity may or may not be sufficiently reduced to enable the jet to fill the tube thereafter. To accomplish this several types of commercial aspirators have baffles or zig-zag exit tubes. While such devices may serve the purpose intended they cause very great head losses and lead to corresponding losses in efficiencies. The much better plan is so to design the tube as to make such provisions unnecessary. That this can be done may be clearly shown by the results in a particular case. A trial run was made of a certain metal aspirator of the Bernoulli type and having an exit tube that expanded very rapidly from the throat and carried an extra zig-zag section of tubing at its lower end. This exit tube was drilled as far as the throat and a 5/16 in. brass rod reaching to the end of the exit tube was inserted. This rod was drilled and reamed in such a way as to conform to necessary conditions outlined above and the zig-zag section again attached. This single change made the aspirator three times as efficient as it was originally. Next the new type of nozzle was substituted for the original, and the efficiency rose to five times its first value. Moreover the aspirator would then operate as well without the zig-zag section as with it.

Further illustrations may serve to explain the nature of the problem and its solution. It was found, for example, when the nozzle holes in the disk, *N*, were cylindrical, or were made like fire-hose nozzles by reaming them from the upper side, that in general the aspirators would not operate without the addition of a short length of rubber tubing. In a particular case the disk had cylindrical holes and was used in what was considered a standard aspirator body, but it would not function without the addition of about 5 cm of tubing. When these holes were reamed on the under side to give them a steam-nozzle form and thereby cause them to produce a spreading jet,[6] not only was the efficiency of the aspirator doubled, but the jet was able to fill the exit tube unassisted and operated well either with or without the use of rubber tubing. In fact this form of nozzle was finally adopted as standard, and when used with an exit tube suitably designed made it possible greatly to reduce the length of the latter. The result was a

[6] For further discussion of the effect of nozzle form treatises on hydraulics should be consulted.

pump that could be used either as an aspirator or as an ordinary hose nipple.

4. *Water Consumption.* The importance of cutting down the water consumption is two-fold. In the first place a large water consumption lessens the effective pressure of the system and therefore at the aspirator itself, particularly where several aspirators are used simultaneously. In the second place either of two widely used aspirators operating on a pressure of 40 lbs. per square inch will use in the neighborhood of 3,000 liters of water in a four-hour laboratory period. Even as cheap as water usually is, such an aspirator used much of the time will require every year an amount of water costing many times as much as the aspirator itself—a fact not generally appreciated.

RESULTS

As stated above, the index of efficiency was arbitrarily taken as the product of the speed of the pump in cubic centimeters per second by the time in second T required to use 1 liter of water. Hence any study of pump efficiency must involve two factors, namely: speed and water consumption. For obtaining the speed there is available the Gaede formula:[7]

$$S = \frac{V}{t} \log_e \frac{(P_1 - P_0)}{(P_2 - P_0)}$$

where S is the speed (e.g., in cc/sec.), V, the volume being evacuated, t, the time between the observations of pressure P_1 and P_2 and P_0 is the minimum pressure attainable.

The tests on efficiency were made by attaching the aspirator to be tested to a water tap provided with a high grade water pressure gauge. The system consisted of a 3400 cc glass bottle, a stop cock, and a closed tube manometer, with the necessary rubber tubing for connections. The observations taken for any test gave the following data: manometer and time readings taken at 10 sec. intervals, mean water pressure, and the time required to pass 5 liters of water.

On the basis of these data time-pressure curves were drawn as shown in Fig. 3. For the purposes of making comparisons three pressure ranges were chosen—from 35 to 20 cm, from 20 to 10 cm, and from 10 to 3 cm. These pressure limits were designated p_1, p_2, p_3, p_4, and the intercepts of the curves on these lines gave the corresponding

[7] Dushman, High Vacuum, p. 58.

values of t_1, t_2, t_3, and t_4. The vapor tension of the water, which varies say from .9 to 1.5 cm under usual temperature conditions, was taken as the value of p_0. The substitution of these values in the Gaede formula enabled one to calculate the aspirator speeds for the various ranges, and the products of these values of S by the corresponding values of T gave the values of the index of efficiency. To illustrate by a concrete example, aspirator No. 32, of the type shown by Fig. 2, gave the data for curve B. For the pressure range of p_1 to p_2 this gives a value for S of 144 cc/sec; for the range p_2 to p_3 a value of 187; and for p_3 to p_4 a value of 219. It required 5.8 sec. to discharge 1 liter of

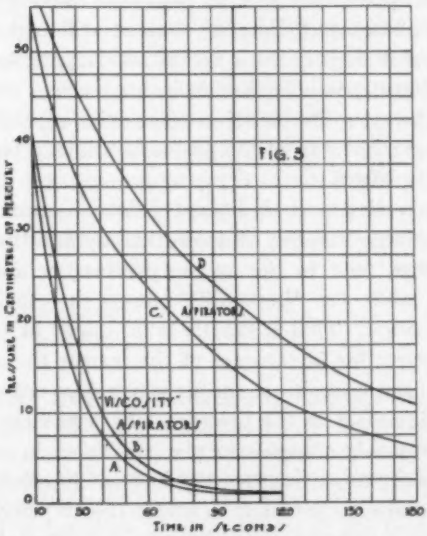

FIG. 3. *Curves showing the relative performances of the new type or "viscosity" aspirators and two widely used aspirators of the "Bernoulli" type.*

water at a pressure of 40 lbs./in.^{2}. The corresponding indices of efficiency are therefore 835, 1094 and 1267. Curve A shows the performance of an aspirator of the same type but having a longer exit tube.

Curves C and D represent the performances under the same conditions of two of the best known and most widely used aspirators of the Bernoulli type. They yield values of 240 and 154, respectively, for the indices of efficiency, one fifth and one seventh respectively that shown by curve B. These two aspirators have practically no jet exposures, very large exit tube diameters, and employ baffles or zig-zag

exit tubes. Their performances, while poor, are exactly what one might expect on the basis of the above theoretical considerations.[8]

A number of tests was made to study the effect of variations in water pressures upon the efficiencies of aspirators. It was found that the increase in speed at higher water pressures more than offsets the disadvantage of higher water consumption, so that the efficiencies tend to increase with water pressure. For "Bernoulli" aspirators this increase is practically linear, and in an increase in water pressure from say 18 to 55 lbs./in.^{2} there was an increase in efficiency of nearly one hundred per cent. For the new type of aspirators the increase was somewhat more rapid, and at from 40 to 45 lbs./in.$^{-2}$ the efficiency was twice as high as at 18 lbs./in.^{2}, but at still higher pressure the efficiency decreases slightly with the increasing pressure. The exact mode of variation depends on the aspirator, as does also the point of maximum efficiency. The relative efficiencies of the two types of aspirators, however, at any of the pressures studied, is practically the same as shown in Fig. 3.

A highly interesting and significant point observed in connection with tests of a large number of aspirators is that the efficiencies of "viscosity" pumps tend to rise as the pressure in the system being exhausted falls while for "Bernoulli" pumps the efficiencies tend to fall with the pressure. That is exactly what might be expected on the theory above given, for in the case of the former the operating force is due largely to the viscosity of the air which does not diminish in the pressure range involved as the pressure falls, while the inertia of the air and the volume it occupies in the exit tube do decrease. This would appear easily to account for the increase in efficiency observed. Bernouilli aspirators are not built to utilize the air viscosity, and even granting that that principle furnishes the true explanation of their action the operating differential pressures must decrease as the pressure falls towards the lower limit fixed by the factors already discussed. One would expect their efficiencies to fall accordingly. In fact the study of the results obtained from a large number of different glass and brass aspirators would appear to lend support to the conclusion that an aspirator is likely to be efficient in proportion to the extent of effective exposure of the jet to the air, assuming a properly designed exit tube. The efficiencies of the aspirators tested ranged between

[8] Incidentally, aspirators No. 32 and No. 6 which gave the data for curves *B* and *D* furnish a striking illustration of the non-dependence of efficiency on size; for while No. 6 is six times as heavy as No. 32, it is only one-seventh as efficient.

the values above given. Only one of the glass ones and just one of the brass were found to have efficiencies of the same order of magnitude as those of the new type pumps above described. It would seem that the general impression that glass aspirators are better than brass must have as its basis the fact that, accidentally or otherwise, glass blowers are likely to leave more jet exposure than is usually provided in aspirators made of metal which are subject to more exact and closer margins in manufacture. It should be added, however, that a casual examination of an aspirator is not likely to lead to a very accurate estimate of its efficiency, for it is found that even slight and unobservable variations in nozzle form or exit tube may be quite sufficient to affect the jet surface or form, or its manner of entraining air in the exit tube to an extent that may cause very large variations in efficiency, and as mentioned above the performances vary widely with the water pressure. It is therefore impossible to say that an aspirator found highly efficient in one situation will be found equally efficient in another. But the efficiency of the new type aspirators is so many fold greater than that of the old that these probable variations in efficiency with conditions become unimportant, and in any case are likely to be as applicable to the one type as the other.

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Doppler effect due to transfers of energy from excited to unexcited atoms? If a Hg atom in the $2p_2$ state (energy 4.9 equivalent volts greater than for the normal state) puts a sodium atom into one of its $2p$ states (energy approximately 2.1 equivalent volts above that of the normal state) there is left over an amount 2.8 equivalent volts which, if it becomes kinetic energy of the two atoms, confers upon the sodium atom a speed $4.3 \cdot 10^5$ cm/sec. If the sodium atoms emitting the *D*-lines in a mixture of Hg and Na irradiated with the light of a Hg arc are moving in random directions with this speed, then owing to Doppler effect the *D*-lines should each appear 0.17A broad and uniformly bright. Sodium vapor of equal density, pure and irradiated by the *D*-lines, emits *D*-lines which are much narrower. These predictions are confirmed by measurements with an echelon and ocular micrometer.—[F. Rasetti (Florence); Nature *118*, p. 47, 1926.]

KARL K. DARROW

Oxidation of metals.—The amount of oxidation suffered by metals in contact with gaseous oxygen is determined for various durations of contact and temperatures of the system, by measuring sometimes the gain in weight of the solid and sometimes the diminution of pressure of the gas. The experiments are performed upon Cu and Cu-Zn alloys of various compositions (95%, 90%, 70% Cu) and there are prior data available for Cu, Fe, Zn, Cd, Al and Ni. Usually the amount of absorbed oxygen at each temperature varies as the square root of the duration of contact, as would be the case if the phenomenon were purely a diffusion of the gas into the solid; but there are exceptions (generally in the sense that the rate of oxidation eventually decreases more rapidly than the square-root law implies) one case being that of copper "activated" by repeated oxidation and reduction; these are attributed to changes occurring in the oxidized part of the solid as time goes on. The amount of oxygen taken up in a given time varies as $\exp(K/T)$ with the temperature T of the system, the constant K being nearly the same for Fe, Ni, Cu (at high temperatures) and the three Cu-Zn alloys. Independent attempts were made to determine the thickness of the oxide film (one would expect this to be a very vague concept if the analogy with a diffusion-process holds good!) from surface-colors and by measuring the conductivity of copper in the form of a thin film. The square of the diminution in conductivity varies as the duration of contact between film and oxygen, which confirms the prior data if the fall in conductivity is proportional to the amount of oxygen absorbed, which the author infers from these data and the observations on surface-colors.—[J. S. Dunn; Proc. Roy. Soc., *A* 111, pp. 203–219; 1926.]

KARL K. DARROW.

Resistances of Au, Zr, Cd, Pt, Ni, Fe, Ag at temperatures from 20° to 1°.34K. —The most important result of an investigation of these resistances by Meissner is, that none of these metals shows any sign of becoming supraconductive down to the lower temperature-limit cited—a conclusion the more valuable, because most of the metal samples were of an exceptional purity and some were tested both as single crystals and as polycrystalline masses. The fact previously observed by Tuyn, that certain samples of Cd do become supraconductive, is attributed to very slight contamination by lead surfaces with which the Cd had been in contact. If this explanation is correct it disposes of the idea that supraconductivity is a quality of extremely pure metals only.—[W. Meissner (Reichsanstalt); ZS. f. Phys., *38*, pp. 647–658; 1926.]

KARL K. DARROW.

A PRECISION LABORATORY METHOD FOR DETERMINING THE MECHANICAL EQUIVALENT OF HEAT

By J. M. Cork

ABSTRACT

An apparatus for determining the mechanical equivalent of heat by measuring the heat developed within a stationary copper cylinder as well as the torque developed by it when placed in a rotating magnetic field, is described. The field strength is made unvarying with angular position by the actual rotation of an electromagnet with slip rings. Radiation loss is reduced to a minimum by immersing the copper cylinder in water contained within a thermos bottle. In addition to the mechanical and heat energy measurements, the equivalent electrical energy is measured at the same time.

The value of the relationship between the thermal energy unit (calorie) and the mechanical energy unit (erg) has now been obtained by a great variety of experimental methods. In most methods the radiation correction has been such an important factor that uncertainties regarding it have made the results invalid. Further in most methods the mechanical torque changes during the course of the experiment so that it is difficult to evaluate the total mechanical energy dissipated. These effects together with the difficulties in the actual measurement of temperatures and forces have made an accurate determination of this relationship something to be obtained only by the most painstaking endeavor.

The mechanical equivalent of heat is of such importance in its relation to our electrical units that any method of obtaining it with accuracy and ease is deemed worth description.

As early as 1870 the possibility of determining this quantity by means of the heat developed within a copper disc rotated in a magnetic field was discussed by M. Voille. In 1898 Baille and Féry[1] described a method for the determination by placing a copper cylinder in the rotating magnetic field produced within an iron ring which was wound with coils carrying polyphase currents. The copper cylinder was prevented from rotating by a counter torque, thus enabling the work to be evaluated. However, if the vector which represents the rotating magnetic field strength varies in magnitude with rotation as is generally the case, then the heat energy will be greater than the mechanical energy. Further in their experiments the energy lost by radiation was

[1] Comptes Rendus, *126*, p. 1494; 1898.

as much or more than that retained so that great inaccuracies were possible.

APPARATUS

The present apparatus is shown in section in Fig. 1. A rotating magnetic field of constant strength is produced by rotating an electromagnet by means of a motor. The exciting current for the electromagnet is supplied from a storage battery by means of slip rings and brushes, *S*. The core of the electromagnet is made of a hollow shell of

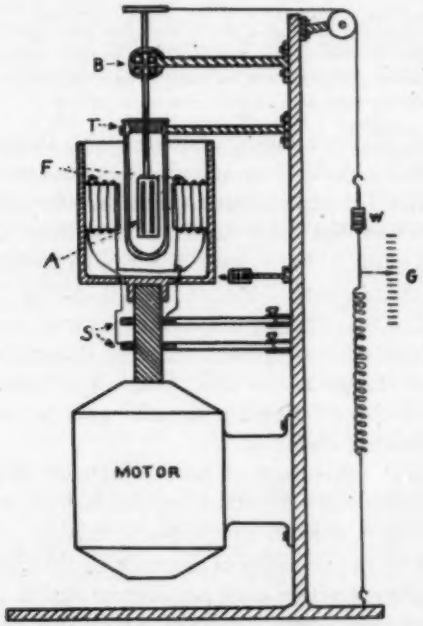

FIG. 1. *Sectional view of rotator.*

iron carrying the pole pieces *F* within. The vertical motor is a shunt wound machine operated from storage battery so that its speed may be maintained at any constant value. Eddy currents are induced within a hollow copper cylindrical armature, *A*, situated symmetrically at the axis of the electromagnet. This cylinder is slotted vertically along the cylindrical surface and is closed at the ends by copper discs with a few perforations for water circulation. Within the copper cylinder is a shell of iron to reduce the reluctance of the magnetic

circuit. The armature is supported by a shaft, a section of which is made of insulating fiber, suspended from two very light ball-bearing units. At the top of the shaft is an accurately turned wheel, at the edge of which a very light silk line is attached for measuring the torque developed. The tension in the thread is measured by the suspended masses in addition to the deflection of a calibrated spring *G*. Using the deflection of the spring as an indicator the torque may be kept very constant at any desired value during a test by controlling the magnetic field strength.

The armature is surrounded by water contained in a straight side thermos flask suspended within the magnetic field. The flask is covered with a thick stopper except for the opening at the center for the supporting shaft. This reduces radiation losses to a very low value. For a temperature of contents 5°C greater than room temperature the rate of cooling was only about .004°C per minute. A revolution counter is mounted so that it may be connected or disconnected at will thus giving the total number of revolutions for any run. All parts are rigidly mounted to a heavy cast iron support. Temperatures are measured by a multiple copper-constantan thermocouple calibrated together with a high resistance galvanometer.

In addition to the mechanical energy and the heat energy measurements, the equivalent electrical energy may be measured at the same time. This is accomplished by a wattmeter which is connected so as to indicate the power supply to the armature of the driving motor. (Since the field of the motor is constantly excited its power may be neglected.) The wattmeter is read with the motor turning at the same speed, both with and without the electromagnet excited. Correction is made for the heating due to the additional armature current with load. Other losses such as windage and frictional losses should thus be

TABLE 1.

Water equiv. of calorimeter, 78.0 g; water added, 377 g; radius of torque wheel, 2.345 cm; room temp., 25.4°C; temp. change corrected for radiation, 3.14°C.

Time	Temp. °C	Revolution counter	Elec. power transformed	Total force	
					ENERGY:
Start, 10:10	24.31	21,643	3.75 watts	326 g	Mech...ergs,..5958.1·10⁷
Stop, 10.36	27.41	33,923			Elec...*joules, 5850
10:38, 40,42	27.43				Heat....calories,1419.3
11:40	27.31				
					*J=4.198·10⁷ergs/calories

accounted for as they depend largely upon the speed of the motor. Table 1 shows a typical record of one trial with the apparatus.

It might be expected that some heat with no accompanying measurable torque would be produced by eddy currents in the silvering of the inner wall of the thermos bottle. Tests for this effect by rotating the electromagnet with the armature removed for a period of an hour showed no result. The greatest inaccuracies in the method as used to date lie in the actual measurement of torque and temperature. The temperature can be read only to about $\pm .01°C$, giving a probable error of $\frac{1}{2}$ of one per cent for a temperature rise of 4°C. In measuring the force there is sufficient friction in the ball bearing units to introduce an uncertainty of as much as 1 gm on a total load of 400 gms. By improving upon these measurements the method should be capable of an accuracy equal to or exceeding any previously described.

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Industrial research laboratories in the United States. The continuing demand for a new edition of Bulletin No. 16 of the National Research Council which contains a list of Research Laboratories in Industrial Establishments of the United States indicates that this list apparently fills a certain need in the industrial world, and the council is confronted with the fact that the material assembled in 1921 is five years out of date and should be revised at this time.

It is undoubtedly true that since 1921 a large number of industrial concerns have established research laboratories. It is also probably true that the 1921 list does not contain a complete roster of the firms which had laboratories at that time. In order that the new list may be as complete as possible an appeal is hereby made to the readers of this Journal who are connected with firms now maintaining research laboratories to send to the council a post-card giving the name and address of the firms with which they are connected. Questionnaires will then be sent to these firms.

The value of such a compilation lies first in its accuracy; second, in its completeness. The completeness of this compilation can be assured only by the cooperation of those in immediate touch with the various laboratories which should be included.

Please address your post-card to the Research Information Service, National Research Council, B & 21st. Streets, Washington, D. C.